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Sequential Power-Dependence Theory

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Existing methods for predicting resource divisions in laboratory exchange networks do not take into account the sequential nature of the experimental setting. We extend network exchange theory by considering sequential exchange. We prove that Sequential Power-Dependence Theory—unlike Power-Dependence Theory and most other exchange theories—has a unique point prediction for resource divisions in every network, and we show that these point predictions fare well in comparison to those from established theories.

Keywords: exchange, power, sequentiality, social networks

1. INTRODUCTION

How well people do in social or economic exchange depends heavily on how they are connected to potential exchange partners. This conclusion draws on a branch of sociology that seeks to answer the question at what ratio two trading parties exchange if they are embedded in a network of exchange relations (Willer, 1999). Willer provides an overview of the theoretical and empirical literature in this field.

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Although there is variation in the precise experimental set-up used by most empirical studies that test theories of network exchange, they mostly use variants of the following experimental procedure.

Subjects are positioned at a networked computer. Subjects occupy different positions in a network and interact in what we call a network exchange game. In the network exchange game, a subject can make offers to other subjects he is connected to in the network (neighbors) as long as he has not already closed a deal with a neighbor. A subject can also accept an offer from a neighbor, or confirm a neighbor's acceptance of an offer he made himself. An offer is a proposed division of 24 profit points. Whenever an offer is accepted and confirmed, the two subjects exchange; that is, they receive the share of the profit they agreed upon. They are removed from the network. Thus, each subject can close a deal with at most one neighbor in the network, which has been called the "one-exchange rule." Typically, this experimental procedure is repeated a number of times with the same subjects occupying the same or different network positions.

Consider a network called the 3-Line (see Fig. 1). A node represents a position and an edge a relation in which exchange might take place. In the 3-Line, B can exchange with A and C, but A and C can exchange only with B. Alternatively, in the 4-Line (see Fig. 1), B and C can exchange with one another or with A and D, respectively, while A and D can only exchange with one actor, namely, B and C, respectively. Table 1 shows an imaginary bargaining sequence for the 4-Line in the experimental setting outlined above.

At the beginning of the sequence, the 4-Line looks like the bottom left structure in Figure 1. The first accepted and confirmed offer is the (10, 14) division proposed by D to C at time 0:16. After their exchange at time 0:21, the only remaining potential exchange is one taking place between A and B. The network now looks like the bottom right structure in Figure 1 and is essentially a dyad with two isolates. McGrimmon and Dilks (2005) call this reduced network a 'decay product.' The exchange between A and B occurs at time 0:42 after a number of offers and counteroffers.

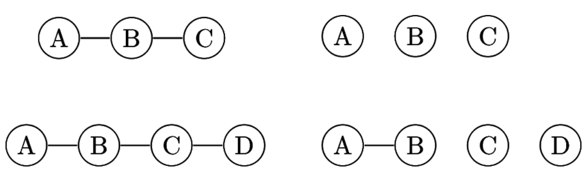

FIGURE 1 Top: 3-Line, before (left) and after (right) B and C exchanged; bottom: 4-Line, before (left) and after (right) C and D exchanged.

TABLE 1 An Imaginary Bargaining Sequence

From	To	Kept	Offered	Type	Time
A	B	16	8	offer	0:07
C	B	17	7	offer	0:08
B	C	15	9	offer	0:12
D	C	10	14	offer	0:16
C	D	14	10	accept	0:19
D	C	10	14	confirm	0:21
C	D	14	10	exchange	0:21
A	B	12	12	offer	0:23
B	A	15	9	offer	0:26
A	B	12	12	offer	0:28
B	A	14	10	offer	0:32
A	B	12	12	offer	0:35
B	A	12	12	accept	0:40
A	B	12	12	confirm	0:42
B	A	12	12	exchange	0:42

Numerous methods have been proposed to predict which subject gets how much in which relation. These methods share one weakness: They implicitly or explicitly assume that all exchanges take place simultaneously. Let's go back to the 4-Line after C and D have exchanged (right in Fig. 1). C and D can now no longer exchange. The only actors that can still exchange after C and D closed their deal are the remaining A and B. The situation is symmetric for A and B: Neither has an alternative relation in which to exchange. They have equal bargaining power. Clearly, they should be predicted to agree on a 12–12 split. Yet, existing methods typically predict that B and C *both* obtain a majority of the 24-point profit pool in their exchanges with A and D. One notable exception is footnote 9 in Markovsky et al. (1988). In this footnote, the authors explicitly make a similar point as we make, namely, that power positions might change if some actors have already finished their exchanges. However, they first note that for small networks the iterative method is not necessary. The example for the 4-Line above illustrates that also in small networks profit splits differ, depending on which pair exchanges first and which one last. In addition, Markovsky et al. (1988) claim that it suffices to recalculate profit splits in reduced networks. In contrast, we argue that profit splits in first exchanges are a direct consequence of what can still be obtained in reduced networks. The anticipation of power loss in the reduced 4-Line network should cause profit splits to be more equal in the unreduced 4-Line. The power B derives from the ability to exclude A has been recognized in past research, but the power A derives from B's potential loss of this ability after a successful exchange between C and D has been neglected.

The lack of attention to sequentiality in existing theoretical models is particularly striking in full-information conditions. In some experiments, information allows subjects to infer only for some of the other subjects whether they have already exchanged. Under such conditions, actors only know that as time goes by, their pool of alternative exchange relations shrinks. For the majority of network exchange experiments, however, all actors see the offers of all the other actors in the network (see McGrimmon and Dilks, 2005, for details). It is then obvious which actors have already closed a deal and which are still in the process of negotiation.

In this article, we attempt to rectify this discrepancy. Using the logic of Power-Dependence Theory (PDT), we provide new predictions that take into account that after two actors have closed a deal, the bargaining power of the remaining actors in the network might have changed due to the fact that two actors are now 'out of the network.' We make separate predictions for the different orders in which non-simultaneous exchanges can take place. For example, for the 4-Line, we have one profit-split prediction for A and B if they exchange first and another one if they exchange second.

We dub the adjusted theory Sequential Power-Dependence Theory (SPDT). It turns out that the sequentiality correction has two additional welcome consequences. We are able to prove that it provides an earnings prediction for every actor in the network, and that each prediction is single-valued.

The next section reviews PDT. In section 3, we introduce SPDT, prove existence and uniqueness, and work out an example. Section 4 assesses SPDT's goodness of fit and compares its performance with that of other theories. The discussion section points out weaknesses and suggests extensions of SPDT's applicability to alternative exchange settings.

2. POWER-DEPENDENCE THEORY

PDT stems from early work on exchange by Richard M. Emerson (1962, 1964, 1972). He defined the power of an actor A over B as equivalent to the dependence of actor B on A. B's dependence on A is the difference between the profit that he earns in exchange with A and the profit he would earn if he would not exchange with A (also known as 'conflict point', 'disagreement point', and 'opportunity costs'). The main thesis is that two parties will exchange at the rate at which A's dependence on B is equal to B's dependence on A. Thus, in a bilateral monopoly, when A and B are the only actors, their conflict points are zero. They are then predicted to divide exchange benefits (or profit pool or surplus) equally. If they do so, A's dependence on B equals half the surplus minus zero and B's dependence on A equals half the surplus minus zero. This is referred to as the equidependence principle. The principle was first applied to exchange networks by Stolte and Emerson (1977) and later by Cook and Emerson (1978). In exchange networks, conflict points are often non-zero, so that surpluses may become divided unevenly. An example is the 3-Line (A–B–C), in which B can exchange with either A or C but not both, and A and C cannot exchange with one another. B's conflict point in exchange with A is what he can get from C, and vice versa. It turns out that there is a unique distribution of the two surpluses at which there is equidependence in both relations, namely, when B gets the entire surplus in either relation. Indeed, B's dependence on A (C) is then the entire surplus minus the entire surplus and C's (A's) dependence on B is zero minus zero. In a special issue of *Social Networks*, dedicated to exchange networks, Cook and Yamagishi (1992) introduce an algorithm for determining equidependent exchange rates in any experimental exchange network. For example, this algorithm generates profit splits 8–16, 8–8, and 16–8 in the 4-Line.[1] How often each exchange relation is used is left unspecified, except that suboptimal relations are predicted to occur rarely. Suboptimal relations are relations that, if used, prevent the maximum number of exchanges in the network from occurring. An example is the B–C relation in the 4-Line, which reduces the total number of exchanges from 2 to 1.

The problem with this method, and with all other existing profit prediction methods, is that it is a simultaneous-play model of sequential exchange: It requires that equidependence is reached in all relations simultaneously, while deals are made sequentially in the network exchange experiment. In exchange relations that are used after others have already exchanged, alternatives may be foregone. Some actors have alternative exchange possibilities only as long as these other neighbors have not already exchanged. For example, in the 4-Line, knowledgeable middle actors are in a hurry, because a dyad is left in which all bargaining power is lost after the other middle actor has exchanged.

[1]Cook and Yamagishi (1992) predict 'waste' in some relations. For example, the 8–8 split implies that actors leave some money on the table, since they have 24 points at their disposal. Yet, the experimental setting described before does not allow participants to waste resources. The method we propose here, SPDT, precludes waste.

One possible defense of PDT (and other theories that do not explicitly take the sequentiality into account) is that the predictions should be considered mere averages over differently timed exchanges in multiple rounds of the experiment. The prediction 8–16 for A and B in the 4-Line could, for example, represent an unweighed combination of 4–20 splits in first exchanges and 12–12 splits in second exchanges. However, this defense undermines the theoretical plausibility: Why would average bargaining outcomes be equidependent while single bargaining outcomes are not?

The SPDT method that we present in the next section takes the sequentiality into account. SPDT has two additional advantages over PDT. There are networks for which a wide range of PDT solutions exist. An example is the Box network (see Fig. 2). Of course everybody obtaining 12 in any exchange is a PDT solution, but also one pair of unconnected actors obtaining $24-x$ and the other two actors obtaining x is a PDT solution. So, any distribution in which two unconnected actors receive the same share and the others obtain the remainder is equidependent. It turns out that a sequential conceptualization of network exchange leads to a unique prediction.

The theory we present here does not need to be considered an extension or adaptation of earlier versions of PDT. What we borrow here is the idea that actors exchange according to an equal gain principle relative to some well-defined conflict points. This corresponds with the Nash solution (Nash 1953) in general bargaining theory (see also Braun and Gautschi 2006; Van Assen 2001: ch. 7 provides an extensive overview of the commonalities between network exchange theories and more general bargaining principles). We define the conflict points based on what two actors can obtain in the network if they as a pair would not exchange first, that is, if one of the other pairs would exchange first. In this manner, we obtain conflict points as a

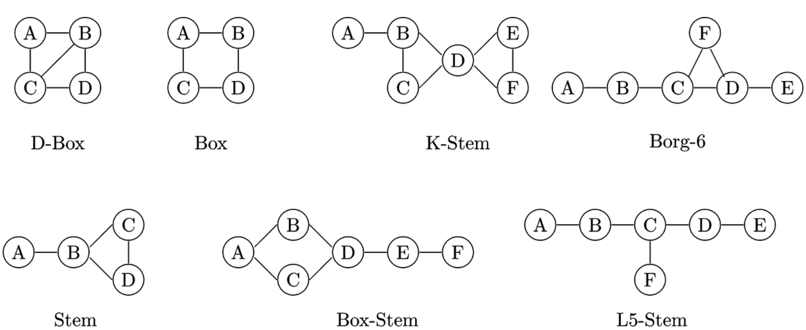

FIGURE 2 Example networks.

consequence of taking the sequentiality of the exchanges seriously. In the next section, we show that just this conceptualization of equidependence alone is sufficient to generate a unique feasible profit distribution for every relation in every network.

3. SEQUENTIAL POWER-DEPENDENCE THEORY

Let $\boldsymbol{X} = (x_{ij})$ be an $n \times n$ symmetric network, with $x_{ij} = x_{ji} = 1$ if actors i and j can exchange, 0 otherwise. Let $N(\boldsymbol{X}) = \{\{i,j\}|x_{ij} = 1\}$ be the set of ties in network $\boldsymbol{X}$, where ties are two-element subsets of the set of all actors, and let $N = |N(\boldsymbol{X})|$ be the cardinality of this set. $\boldsymbol{X}\backslash\{i,j\}$ is the network $\boldsymbol{X}$ with the ith and jth column and row deleted, i.e., with i and j and all their relations removed from the network.

$v_i(\boldsymbol{X})$ is the value of network $\boldsymbol{X}$ to actor i, which we define as i's expected value across all his relations. If i is expected to earn all of the 24 points, $v_i(\boldsymbol{X}) = 1$, while if he is expected to earn 12 points on average, $v_i(\boldsymbol{X}) = 1/2$.[2] Let p_{ij} be the probability that i and j are the first to exchange. Because only actors that have a link can exchange, $p_{ij} = 0$ if $\{i,j\} \notin N(\boldsymbol{X})$. Because exactly one pair will exchange first, $\sum_i \sum_{j>i} p_{ij} = 1$. We denote $p^{ij}_{kl} = p_{kl}/(1 - p_{ij})$ as the conditional probability that k and l are the first to exchange if i and j are not. We define $p^{ij}_{kl} = 0$ if $p_{ij} = 1$, which corresponds below with i and j dividing the profit pool equally if they have to exchange with each other, which should be the case because they both do not have an alternative. It follows that $v_i(\boldsymbol{X}) = v_j(\boldsymbol{X}) = 1/2$ if $n = 2$ and $x_{ij} = x_{ji} = 1$ (a dyad). Furthermore, $v_i(\boldsymbol{X}) = 0$ for any actor i without any ties. We define s^i_{ij} as the share an actor i obtains if he is involved in the first exchange with actor j. Because the profit pool is always distributed in its entirety, $s^j_{ij} = 1 - s^i_{ij}$. If i is not involved in the first exchange, but instead j and k are, then he receives what he can obtain in $\boldsymbol{X}\backslash\{j,k\}$. For our example of the 4-Line, in the situation after C and D have exchanged, right in Figure 1, the 4-Line has been transformed into a dyad. A and B thus earn 1/2 each. Alternatively, for the 3-Line (see also Fig. 1), A will earn 0 after B and C have exchanged.

The value of a network to an actor i can thus be recursively defined as

$$v_i(\boldsymbol{X}) = \sum_{j \neq i} p_{ij} s^i_{ij} + \sum_{j \neq i} \sum_{k \neq i, k > j} p_{jk} v_i(\boldsymbol{X}\backslash\{j,k\}). \quad (1)$$

[2]We transfer the rather arbitrary profit pool of 24 points here to a 0–1 scale via the value function. Our theory does not depend on the number of points to be divided, and it assumes that the pie to be divided, can be divided in a continuous manner.

where the values for p_{ij} are given. Now, suppose that for a certain network size n, we know each value v_i for each actor i of each network of smaller size. Then, in order to be able to calculate the values of a network of size n for all actors in that network, what remains to be done is computing the s^i_{ij}. Consider the case that i and j exchange first. We now apply the equidependence principle that each actor gets his expected value of the conflict point plus half of the remainder of the profit pool (this remainder might be a negative amount). More precisely, i's value of the conflict point in an exchange with j is what i would have gotten if i and j would not have been the first to exchange. This is what i gets in the case of each of the other relations exchanging first, weighted by the corresponding probabilities of first exchange.[3] In a formula, i's expected value if he does not start to exchange with j equals

$$\sum_{k \neq i,j} p^{ij}_{ik} s^i_{ik} + \sum_{k \neq i,j} \sum_{l \neq i, l > k} p^{ij}_{kl} v_i(\boldsymbol{X} \backslash \{k, l\}), \tag{2}$$

in which the first term concerns the sum of shares for i if i exchanges with someone else in the first opportunity and the second term concerns i's expected values when not exchanging first. A similar formula holds for j and the equidependence principle then implies that the difference between the shares in a first exchange between i and j should be the same as the difference between their expected values if they would not exchange with each other. Thus,

$$\begin{aligned} s^i_{ij} - s^j_{ij} = &\sum_{k \neq i,j} p^{ij}_{ik}(s^i_{ik} - v_j(\boldsymbol{X} \backslash \{i, k\})) + \sum_{k \neq i,j} p^{ij}_{jk}(v_i(\boldsymbol{X} \backslash \{j, k\}) - s^j_{jk}) \\ &+ \sum_{k \neq i,j} \sum_{l \neq i,j, l > k} p^{ij}_{kl}(v_i(\boldsymbol{X} \backslash \{k, l\}) - v_j(\boldsymbol{X} \backslash \{k, l\})), \end{aligned} \tag{3}$$

or

$$\begin{aligned} 2s^i_{ij} = 1 &+ \sum_{k \neq i,j} p^{ij}_{ik}(s^i_{ik} - v_j(\boldsymbol{X} \backslash \{i, k\})) + \sum_{k \neq i,j} p^{ij}_{jk}(v_i(\boldsymbol{X} \backslash \{j, k\}) - s^j_{jk}) \\ &+ \sum_{k \neq i,j} \sum_{l \neq i,j, l > k} p^{ij}_{kl}(v_i(\boldsymbol{X} \backslash \{k, l\}) - v_j(\boldsymbol{X} \backslash \{k, l\})), \end{aligned} \tag{4}$$

The right-hand side of Equation (3) consists of three parts. The first part indicates the difference between how much i and j would obtain if i and an other actor than j were the first to exchange. The second

[3]Cook and Yamagishi (1992) assume an actor's conflict point to be the maximum he can earn in any of his other relations, while we use the expected value using the probabilities of the alternatives.

part indicates the difference between what i and j would obtain if j and an other actor than i were the first to exchange. The third part indicates the difference between what i and j would obtain if two other actors exchanged first. The difference between the shares of i and j when they exchange first is assumed equal to the difference in the expected shares of i and j when they do not exchange first. The small rearrangement in equation (4) uses the requirement that the proportions the two actors obtain sum to 1. A similar equation holds for every exchange relation $\{i,j\}$ in the network $\boldsymbol{X}$.

Consider Equation (3) for the two potential exchange in the 3-Line in Figure 1:

$$s^A_{AB} - s^B_{AB} = 0 - s^B_{BC} \quad \text{and} \tag{5}$$

$$s^B_{BC} - s^C_{BC} = s^B_{AB} - 0. \tag{6}$$

Equation (5) equates the difference between what A and B would obtain if they exchanged first with the difference between what A and B would obtain if B and C exchanged first. Equation (6) equates the difference between what B and C would obtain if they exchanged first with the difference between what B and C would obtain if A and B exchanged first. The zeros arise because A and C do not obtain anything if, respectively, B and C or A and B exchange first. Taking into account that the shares of two actors adds up to 1 we can rearrange the two equations above to there respective equations corresponding to Equation (4):

$$2s^A_{AB} = 1 - s^B_{BC} \quad \text{and} \tag{7}$$

$$2s^B_{BC} = 2 - s^A_{AB}. \tag{8}$$

This system of two equations in two unknowns is easily solved and it follows that $s^A_{AB} = 0$ and $s^B_{BC} = 1$. This implies that B obtains 1 whatever relation exchanges, while B and C obtain 0 if they are involved in the exchange but also if they are not involved. Consequently, the value of their network position equals 0 for A and C, and 1 for the middle actor B.

Switching to the 4-Line, things become a bit more complicated. Now, for the exchange between A and B Equation (3) is

$$s^A_{AB} - s^B_{AB} = \frac{1}{2}(0 - s^B_{BC}) + \frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}\right), \tag{9}$$

because there are two other possible ties. If B and C exchange first, A will be excluded and obtains 0. If C and D exchange first, the dyad AB is the decay product and A and B will both obtain 1/2. Because B and C have equivalent positions, they will obtain both 1/2 if they are the first to exchange. This implies that the equation above can be solved directly and that A obtains 3/8 in a first exchange, while B obtains 5/8. Therefore, the value for A of this network equals the weighted sum of A's share when A and B exchange first, A's share when B and C exchange first, and A's share when C and D exchange first: $1/3 \times 3/8 + 1/3 \times 0 + 1/3 \times 1/2 = 7/24$, assuming that all ties are equally likely as a first exchange. Similarly, the value for B and C equals 13/24 in the 4-Line, while the value for D is again 7/24.

In general, a network with N ties will lead to N equations with N unknown variables s_{ij}^{i} using the known expected values of actors in smaller networks. The corresponding system of linear equations $\boldsymbol{As} = \boldsymbol{c}$ has $\boldsymbol{A}$ as $N \times N$ matrix of coefficients and $\boldsymbol{c}$ as vector of constants. For any network, the following theorem holds for these N equations.

Theorem 1. *For parameters p_{ij} and given network values in smaller networks $v_i(\boldsymbol{X} \backslash \{i,j\})$, the system $\boldsymbol{As} = \boldsymbol{c}$ always has a unique solution, and this solution satisfies $0 \leq s_{ij}^{i} \leq 1$ for all $\{i,j\}$ in $N(\boldsymbol{X})$.*

Proof. The proof is provided in the Appendix.

Theorem 1, thus, indicates that the set of equations that have to be solved to obtain the SPDT exchange values has a unique solution. The proof indeed establishes that the parameters of the equations ensure that none of the equations is a linear combination of the others, which is enough for the solution to exist and to be unique. The second part of the proof shows that the solution is also always in the correct range; that is, all the predicted s_{ij}^{i} are between 0 and 1. For any nonreduced or reduced network and for any tie in the remaining network, we have a unique prediction for how two actors will divide what they have to divide. Using the probabilities for first exchange p_{ij}, one can then also obtain each actor's expected value in a given network.

As a last example, we take a detailed look at the so-called 'Box-Stem' (see Fig. 2). This network has six actors, A through F, and six ties: $N(\boldsymbol{X}) = \{\{A,B\}, \{A,C\}, \{B,D\}, \{C,D\}, \{D,E\}, \{E,F\}\}$. We assume we have already calculated the values of networks with 4 actors. If A and B exchange first they obtain s_{AB}^{A} and $s_{AB}^{B} = 1 - s_{AB}^{A}$. Then a 4-Line remains in which actors C through F expect to earn 7/24, 13/24, 13/24, and 7/24 in that order as we have explained above. These are the six entries in the first row of Table 2. We repeat this procedure for all other rows in a similar manner. The smaller subnetworks that we find after an exchange are the Box, the 3-Line, the dyad, and the

TABLE 2 Profit matrix for the Box-Stem

First exchange	A	B	C	D	E	F
AB	s_{AB}^{A}	$1-s_{AB}^{A}$	7/24	13/24	13/24	7/24
AC	s_{AC}^{A}	7/24	$1-s_{AC}^{A}$	13/24	13/24	7/24
BD	1/2	s_{BD}^{B}	1/2	$1-s_{BD}^{B}$	1/2	1/2
CD	1/2	1/2	s_{CD}^{C}	$1-s_{CD}^{C}$	1/2	1/2
DE	1	0	0	s_{DE}^{D}	$1-s_{DE}^{D}$	0
EF	1/2	1/2	1/2	1/2	s_{EF}^{E}	$1-s_{EF}^{E}$

isolate. Except for the Box, we have already explained the expected values for all these networks. In the Box the value for everyone is 1/2 because everyone has an equivalent position. This leads to the following list of exchange possibilities in Table 2 in which the entry in, for example, column A and row AB represents the expected value for A if A and B would exchange first.

Now we derive the six equidependence equations. We assume for this example that all relations are equally likely to be the first to be used (the existence and uniqueness of the solution do not depend on that assumption). For the exchange between A and B should hold (subtracting B's expected values from A's in rows 2 through 6 above)

$$2s_{AB}^{A} = 1 + \frac{1}{5}\left(\left(s_{AC}^{A} - \frac{7}{24}\right) + \left(\frac{1}{2} - s_{BD}^{B}\right) + \left(\frac{1}{2} - \frac{1}{2}\right) + (1-0) + \left(\frac{1}{2} - \frac{1}{2}\right)\right) \tag{10}$$

or

$$2s_{AB}^{A} - \frac{1}{5}s_{AC}^{A} + \frac{1}{5}s_{BD}^{B} = 1\frac{29}{120}. \tag{11}$$

Similarly, the other five equations can be derived. Then the solution to this system of six linear equations in six unknowns can be calculated: $s_{AB}^{A} = 0.65$, $s_{AC}^{A} = 0.65$, $s_{BD}^{B} = 0.39$, $s_{CD}^{C} = 0.39$, $s_{DE}^{D} = 0.51$, and $s_{EF}^{E} = 0.60$. These values eliminate all unknowns from Table 2, so that the weighted column averages are now the expected profit shares of the six actors. Since we assumed that all dyads were equally likely to be first, all the weights (p_{ij}) are equal to 1/6. For actor A we have

$$v_A = \frac{1}{6}(0.65 + 0.65 + 0.5 + 0.5 + 1 + 0.5) = 0.63. \tag{12}$$

Similarly, we obtain $v_B = 0.34$, $v_C = 0.34$, $v_D = 0.55$, $v_E = 0.53$, and $v_F = 0.33$ for the other actors.

Alternatively, we can compute average earnings per relation, that is, how two actors in any particular relation on average split their profit pool. To obtain this number for A and B in the Box-Stem we cannot simply average columns A and B. Some of this profit is not earned in the A–B relation. We must weigh each entry by the probability that it is earned in the A–B relation. For example, with 1/6 chance A and B exchange first and A earns 0.65 in his relation with B. With another 1/6 chance, he earns 1/2 in his relation with B, namely when C and D exchange first. With 1/12 chance, he earns 1 in his relation with B, namely when D and E exchange first and A decides to exchange with B rather than C. Last, with 1/12 chance he earns 1/2 in exchange with B, namely when the Box remains after E and F exchange first and A and B indeed decide to exchange. Thus, A and B are expected to exchange with probability $1/6 + 1/6 + 1/12 + 1/12 = 1/2$. To assess the expected profit for A in the A–B relation, we have to calculate the conditional probabilities of each of the four events above. Hence, given that A and B exchange, A earns 0.65 with probability $1/6/1/2 = 1/3$. With probability $(1/6 + 1/12)/(1/2) = 1/2$ he earns 1/2, and with probability $(1/12)/(1/2) = 1/6$ he earns 1 in the A–B relation. This yields an average expected profit of 0.63 for A given that he exchanges with B. These relation earnings are the predictions we use in the next section to assess the empirical performance of SPDT.

4. EMPIRICAL PERFORMANCE

Table 3 compares SPDT's predictions with observed predictions reported in Willer and Emanuelson (2005) and with those from three other methods, including PDT. The value difference with SPDT indicates the improvement in fit resulting from taking sequentiality into account. Another is EVT, the latest version of which was presented in Friedkin (1995). The last is Elementary theory (ET), which Willer and Emanuelson (2005) found to empirically outperform the nine other theories they considered. The predictions are the average earnings of the first-listed actors in the specified dyads.

SPDT with equal probabilities of first exchange (SPDT-Equal) does worse than the empirically most competitive method ET. It has four predictions that are more than 1 point off the observed value and the error, measured by the sum of absolute deviations, is higher, 12.11 instead of 3.45. Striking is the bad fit for the B–A relation in the K-Stem. SPDT-Equal predicts actor B to earn a greater share than B does in the laboratory. One could investigate whether the deviation is due to the equiprobability assumption. Perhaps this tie or other ties are more or less often used in first exchange in the laboratory. Still, SPDT-Equal ranks among the better fitting theories that Willer and

TABLE 3 Comparison of point predictions with other methods (ED and PDT are taken from Willer and Emanuelson (2005)) EVT is own recalculation based on Dogan et al. (2007) implementation of EVT

Network	Dyad	ET	PDT	EVT	SPDT-Equal	SPDT-Random	OBS
4-Line	B–A	13.50	16.00	16.04	13.50	13.00	13.58
Stem	B–A	14.40	18.00	18.43	14.57	14.94	14.41
D-Box	B–A	12.90	12.00	15.13	13.00	13.20	12.80
K-Stem	B–A	14.50	20.00	18.44	18.01	17.83	13.69
Borg-6	B–A	13.50	18.00	16.21	14.75	14.96	14.02
	D–E	14.40	18.00	18.36	16.49	16.12	14.52
L5-Stem	B–A	13.20	16.00	15.26	12.89	12.98	12.91
	C–F	14.30	16.00	17.60	14.75	13.60	13.72
Box-Stem	A–B	12.60	12.00	14.13	15.17	14.08	12.71
	D–B	12.60	12.00	14.13	13.62	12.73	12.82
	E–F	13.30	18.00	15.86	13.03	13.14	12.69
Sum of errors		3.45	32.79	31.72	12.11	10.39	

The table reports the profit the first-mentioned actor earns. ET = Elementary Theory (Willer and Anderson, 1981; Willer and Emanuelson, 2005), PDT = Power-Dependence Theory (Cook and Yamagishi, 1992); EVT = Expected Value Theory (Friedkin, 1995); SPDT-Equal = Sequential Power-Dependence Theory with each tie equally likely to be the first tie to exchange; SPDT-Random: SPDT with the likelihood for the first tie actors randomly seeking neighbors (see next section); OBS = the average observed value in all but the first periods of the laboratory experiment.

Emanuelson (2005) compare (e.g., Bienenstock and Bonacich, 1992; Yamaguchi, 1996; Burke, 1997; Braun and Gautschi, 2006).

SPDT-Equal outperforms PDT as well as EVT. PDT has a total error of 32.79, while EVT has a total error of 31.72. This suggests that the sequentiality correction is not only theoretically but also empirically worthwhile. This correction is in 9 out of 11 cases in the egalitarian direction; the high-power actor earns less under SPDT-Equal rules than under PDT rules. The sequentiality correction is sometimes too weak, and never too strong, for the investigated networks. As the figures show, actors in more advantageous positions in the network usually earn in the lab less or about as much as SPDT-Equal predicts. The overestimation would be slightly reduced if we had taken into account that the most unequal deal subjects in the lab can strike is 23–1, and not 24–0.[4]

[4]Also in strong-power networks such as the 3-Line, SPDT predicts too extreme values for the exchanges. A clear candidate for explaining this overestimation of earning differences is by including some form of inequality aversion, which is a well-known adaptation of the standard model to understand behavior in experimental settings (see, e.g., Fehr and Gintis 2007, for a recent overview).

Even though we have eliminated the problems of simultaneity and non-uniqueness, the necessity of an assumption on the likelihood that an exchange relation is used first for generating an actor's expected value remains. So far we have used the default assumption of equal probabilities of first exchange. The SPDT predictions in Table 3 are based on this assumption. We thus predict what we would observe under the equidependence rule in a laboratory experiment in which pairs of connected actors are randomly selected and forced to exchange. In actual laboratory experiments, subjects may exchange in some relations more often than in others.

In Table 3, we also list SPDT predictions using a random seek mechanism in which actors choose randomly between their partners and if some pairs of two connected actors choose each other, one pair is chosen again randomly to exchange first (SPDT-Random). If no pair of actors choose each other they just try again. Using these alternative probabilities we obtain slightly different predictions as one can see in Table 3.[5] The fit with the random seek probabilities is a bit better for these networks although not for all relationships. The overall improvement comes from the fact that actors with few ties (especially those with only one tie) find partners with a higher likelihood. The smaller probability of being excluded increases their bargaining power and makes them obtain a bit more and their neighbors a bit less. Nevertheless, the differences between SPDT-Equal and SPDT-Random predictions are mostly small. We experimented with more variants of probabilities also including the probabilities on which Friedkin's (1995) EVT is built, but it seems that our predictions do not strongly depend on the precise values of the probabilities.

Although ET still predicts better than SPDT in the evidence at hand, we do not consider this a final defeat for SPDT, because it is unclear how representative the set of networks in Table 3 is. A critical test between SPDT and ET would involve a set of carefully chosen uninvestigated networks for which predictions between the theories vary maximally. As long as such a test is not available, we feel that SPDT deserves the benefit of the doubt. In parsimony it outclasses ET, unambiguously deduced from a smaller set of theoretical axioms, and guaranteeing existence and uniqueness of predicted values in all possible networks.

[5]Predicted values of first exchanges as well as values of network positions for all actors are available from the authors for all networks with eight or less actors. Predictions are available for equal as well as these randomized probabilities of first exchange and also for starting probabilities that exclude some ties as discussed in the discussion section.

5. DISCUSSION

Most theories for predicting network exchange assume simultaneous exchange and do not predict unique resource divisions for each dyad in a network. We introduced Sequential Power-Dependence Theory that assumes sequential exchange and does predict unique resource divisions for each dyad in a network. SPDT thus constitutes a worthwhile theoretical and empirical addition to existing network exchange theories. Although the calculative procedure becomes laborious for larger networks due to the recursive definition, the baseline assumptions are few and they are straightforward. We have proven that the theory provides a unique prediction for exchange outcomes given any set of probabilities with which ties exchange first.

SPDT was found empirically competitive with existing methods using the most straightforward assumption that all ties are equiprobable in first exchange. That this is an untenable assumption for some networks is known from experimental data. Fortunately, we found our profit predictions to vary little across probability methods. Even with equiprobability in the first exchange, we strongly improved on most network exchange theories.

We did not provide explicit theory on what the probabilities of first exchange should be, on purpose. Several types of methods for generating such probabilities have been proposed in the literature, and SPDT generates unique profit predictions for every network using any of these probability methods. However, the existing methods are not based on an assumption of rational action. They do not specify probabilities with which actors maximize earnings. Yet many exchange theories, including SPDT, do employ the notion of a rational, profit-maximizing actor. This theoretical inconsistency is dissatisfying. Finding optimal probabilities is therefore an important task. It is also a difficult task, as we have argued here. It requires the specification of a game that resembles the experimental exchange setting used so far and in which actors strategically choose probabilities of exchange seek. Ideally, one might identify also a unique set of optimal probabilities in that game, but probably there are many rational solutions for that problem. Outcomes of such a game might depend on very subtle changes in experimental settings and, therefore, might be different for various experiments done in the past. Furthermore, the technical derivation of such solutions is complex and laborious and is beyond the scope of this article.

Considerations about the probabilities should also take into account that, in some relations, actors never wish to exchange. Such relations are often called 'breaks' (Markovsky et al., 1988; see also Simpson and

Willer, 1999; Girard and Borch, 2003). The classic example is the T-network in which a dyad is linked to the middle actor of a 3-Line. If the relation between the dyad and the 3-Line did not exist, the middle actor in the 3-Line would earn 1 and the actors in the dyad 1/2. In the T-network they must split 1, earning less. For any positive probability of exchange in the relation that connects the dyad to the middle actor of the 3-Line, the two actors involved in this link both lose as a result of this link. Also, the predicted expected values from our model give rise to situations in which actors who have to exchange first would have preferred not to have had this tie. However, identifying those ties again depends on the precise setup of the exchange setting. Still, if one would like to change some probabilities to zero, this does not change the fact that our method produces unique predictions.

A strong test of our theory could come from reanalyzing some existing data on network exchange in settings in which actors can observe which other deals are closed and for which the order in which deals were made was recorded. One could test predictions from SPDT-Equal about differences in exchange in specific dyads depending on the timing of the exchange in this dyad. Does the second dyad to exchange in the 4-Line exchange at a more equal rate than the first dyad does? McGrimmon and Dilks (2005) provide an experiment using the 7-Line that provides data that are suitable to test also our theory. The results as presented in the paper at least seem in correspondence with what our theory would predict when comparing earlier with later exchanges in the 7-Line. Alternatively, one could test how well the theory performs under the observed probabilities of first exchange. Clearly such a test would not provide a fair comparison to other theories for network exchange, but it would give an indication of how well the equidependence principle fared if one knew the likelihoods of first exchanges.

The theory can also be extended in scope to exchange settings other than the one that is usually studied in the laboratory. The application to exchange settings in which actors can exchange more than once or have multiple connections to the same neighbor (Willer, 1999: ch. 8) is straightforward. We simply add these relations to the system of linear equations. Exchange can also be inclusive rather than exclusive or involve profit pools of unequal size (Willer, 1999: ch. 8). Making SPDT applicable to these exchange settings can be done by changing the iterative procedure. For example, if actors can have more exchanges per time period, one cannot just remove two complete actors from the network after an exchange, but one can remove the relation. After that, the remaining network with exchange possibilities needs to be updated accordingly. The recursive procedure requires more steps and proving that one obtains again a unique solution is far from straightforward. We leave these challenges to future research.

A. PROOF OF THEOREM 1

The rows and columns of the matrix $\boldsymbol{A}$ are labelled with double indices ij referring to the dyads $\{i,j\}$ with $i<j$. Thus $a_{ij,kl}$ refers to the coefficient before the variable s_{kl}^{k} in the equation belonging to $\{i,j\}$ exchanging first. In equation (3) one can see that the diagonal elements are all $a_{ij,ij}=2$. The off-diagonal elements $a_{ij,kl}$ are equal to 0, p_{kl}^{ij}, or $-p_{kl}^{ij}$. Namely, 0 if $k\neq i$, $k\neq j$, $l\neq i$, and $l\neq j$; p_{kl}^{ij} if $k=j$ or $l=i$; and $-p_{kl}^{ij}$ if $k=i$ or $l=j$. Now one can obtain an upperbound for the sum of the absolute values of the off-diagonal coefficients in each row $\{i,j\}$:

$$\begin{aligned}\sum_{\{k,l\}\neq\{i,j\}}|a_{ij,kl}| &\leq \sum_{\{k,l\}\neq\{i,j\}} p_{kl}^{ij} = \sum_{\{k,l\}\neq\{i,j\}} p_{kl}/(1-p_{ij})\\ &= (1-p_{ij})/(1-p_{ij})\\ &= 1 < 2 = |a_{ij,ij}|.\end{aligned}$$

In words, for every row, the sum of the absolute values of the off-diagonal elements is smaller than the absolute value of the diagonal element. This property of matrices is referred to as (strict) diagonal dominance. The Levy-Desplanques theorem (see, e.g., Horn and Johnson, 1994: 302) states that diagonally dominant matrices are invertible. Therefore, $\boldsymbol{As}=\boldsymbol{c}$ has a unique solution.

We know now that there is a unique solution, but this is not sufficient, because the only feasible values for all s_{ij}^{i} lie in the interval [0,1], because the values represent the proportions actors obtain from the profit pool. Strict diagonal dominance is also a sufficient condition for the so-called Jacobi method to converge (see, e.g., Varga, 1962). The Jacobi method is an iterative procedure to solve sets of linear equations. Using this, we can prove the existence of a feasible solution by induction. For $n=1,2$ the solution is feasible, because $s_{ij}^{i}=0$ for isolates and $s_{ij}^{i}=s_{ij}^{j}=1/2$ for two actors in the dyad. Now suppose $n>2$ and the solutions are feasible for all networks of size smaller than n. In the Jacobi method, one expresses an entry of the solution vector in the current iteration $t>0$, $s_{ij}^{i,t}$, as a function of the other entries in the previous iteration $t-1$, $s_{kl}^{k,t-1}$. Rewriting Equation (3) accordingly we obtain:

$$2s_{ij}^{i,t}-1=\sum_{k\neq i,k\neq j}p_{ik}^{ij}\left(s_{ik}^{i,t-1}-v_j(\boldsymbol{X}\backslash\{i,k\})\right)+\sum_{k\neq i,k\neq j}p_{jk}^{ij}\left(v_i(\boldsymbol{X}\backslash\{j,k\})-s_{jk}^{j,t-1}\right)$$
$$+\sum_{k\neq i,k\neq j}\sum_{l\neq i,l\neq j,l>k}p_{kl}^{ij}\left(v_i(\boldsymbol{X}\backslash\{k,l\})-v_j(\boldsymbol{X}\backslash\{k,l\})\right). \quad (13)$$

One then guesses an initial solution, e.g., $\boldsymbol{s}^0=1/2$, so that for $t=0$, we have feasibility. Now suppose $s_{ij}^{i,t-1}$ is feasible for all $\{i,j\}$. Because of feasibility of both v_i in smaller networks and $s_{ij}^{i,t-1}$ for all $\{i,j\}$ and $\Sigma_{\{k,l\}}p_{kl}^{ij}=1$, the right-hand side of Equation (4) is now a weighted mean of elements that are all in the interval $[-1,1]$, and, therefore, the right-hand side of Equation (4) has itself a value in the interval $[-1,1]$. Therefore $-1\leq 2s_{ij}^{i,t}-1\leq 1$, which is equivalent to $0\leq s_{ij}^{i,t}\leq 1$. Thus, $s_{ij}^{i,t}$ is an element of [0,1] and also feasible. Solutions are therefore feasible for any n, which proves Theorem 1.

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