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Dynamic exchange networks

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Abstract

Considering the theoretical and empirical untenability of static exchange networks, researchers have asked how exchange outcomes change when links are added or deleted. The present paper assesses the validity of seemingly sensible propositions concerning the effects of adding and deleting a link on (i) the payoffs of the actors in the link, (ii) the payoffs of actors in neighboring links and (iii) the variance of payoffs in the exchange network. The propositions were examined by applying expected value theory (EVT) to all 13,597 networks up to size 8. All propositions were falsified. Some falsifications of propositions could be attributed to EVTs prediction that actors use sub-optimal exchange relations. Since other well-known theories of exchange, like power-dependence theory and network exchange theory, also predict that actors use sub-optimal relations, these results are robust to selection of the theory of exchange.

Keywords: Exchange networks; Network evolution

1. Introduction

Research on exchange networks has mainly focused on the effects of network structure on outcomes in bilateral exchange (e.g., Cook and Emerson, 1978; Molm, 1997; Willer, 1999). Network structure determines which actors can exchange with whom, where a link signifies a bilateral exchange relation. An exchange relation usually constitutes, as in the present paper, the opportunity to split a common resource pool of 24 points. Successful exchange occurs if two connected actors can agree on a division. There is an upper limit to the number of exchanges that an actor can engage in. Usually this limit is one (the so-called "one-exchange rule"), as also in this paper. It is through this limit that network structure becomes relevant for how resource pools are split. It gives certain actors a credible threat in the bargaining process, namely to turn

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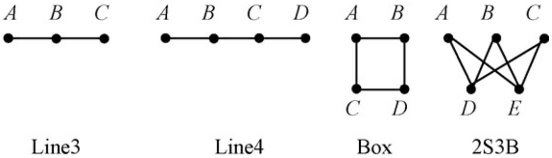

Fig. 1. Some exchange networks.

to an alternative exchange partner, thereby possibly excluding the current exchange partner and leaving him empty-handed. The main result of both theoretical and empirical research on exchange networks is that network structure has a huge impact on what actors earn. Examples of exchange networks are presented in Fig. 1.

In some bilateral exchange relations, actors obtain equally profitable exchange outcomes (Bonacich and Bienenstock, 1995,1997), for example, all actors in the Box (Fig. 1). In other bilateral exchange relations, one actor has a slight advantage over the other actor in the relation, e.g. *B* (*C*) in the Line4 (Fig. 1) obtains a larger exchange outcome than *A* (*D*) in their exchange relation (e.g., Willer, 1999). Finally, in some networks, some actors do have a clear advantage and obtain most of the profit as opposed to other actors who gain almost nothing, e.g., *B* in the Line3 (Fig. 1) obtains most of the profit in his exchange with either *A* or *C* (e.g., Willer, 1999), and *D* and *E* obtain most of the profit in their exchanges in the 2S3B (Fig. 1) network (Willer and Willer, 2000; Corominas-Bosch, 2004).

What the examples make clear is that a single link can be the difference between earning almost nothing, half the resource pool, and almost everything. Even if changing one's links is rather costly, an actor will attempt to do so, e.g., by adding a beneficial exchange relation to his network. For example, if *A* and *C* in the Line3 are both interested in the car *B* has to sell, it is in both *A*s and *C*s advantage to search for an additional seller of a car in order to avoid being exploited by *B*. And, if the Line4 represents a friendship network, *A* (*D*) might be tempted to look for another friend because if his friend *B* (*C*) goes out together or plays tennis with *C* (*D*), *A* (*B*) is excluded.

Benefits of additional links make network dynamics likely, especially when the benefits are large. Not only such theoretical considerations imply network change, also exchange networks (buyer–seller networks, friendship networks, or other social structures that can be considered exchange networks) outside the laboratory are observed to be dynamic. Traders do search for alternative exchange partners to improve their bargaining position in their current exchange relation. People do look for new friends if they feel they put a lot into the friendship and receive little in return. Considering the theoretical and empirical untenability of stasis, a key question becomes how profits change when links are added or deleted.

Yet the question of marginal benefits of links has received little attention in the literature (Kollock, 1994). Only a handful of theoretical studies (Leik, 1991,1992; Willer and Willer, 2000; Bonacich, 2001,2004) and not a single empirical study on dynamic exchange networks has been carried out. The neglect of dynamic exchange networks is particularly striking in light of the very extensive experimental investigations of static exchange networks (Willer and Willer, 2000, p. 252). In these experiments, static networks are exogenously determined by the experimenters. By fixing the network, one of the most powerful forms of strategy to enhance outcomes of exchange is ignored: negotiating changes in the network itself (Leik, 1992, p. 309). Therefore, a desirable step in expanding theory on network exchange is to consider an actor's potential to manipulate (i.e., to delete and add) the links themselves, thereby enhancing his bargaining power in subsequent exchanges, and thus, indirectly increasing his expected payoff.

A few studies have sought answers to the question how profits change when links are added or deleted. Leik (1991,1992) considers exchange networks in which a manipulator is randomly appointed who can propose link changes. Leik derives various propositions on what types of link changes will occur and how these changes will affect payoffs or power differentials in the network, i.e., exchange outcomes at the macro-level. Willer and Willer (2000) assume that actors themselves add and delete links. They evaluate Leik's propositions and add some to them that concern the effects of a change in one actor's link(s) on his own and others' payoffs. Many of their propositions will be referred to in Section 3.

The propositions in Leik (1992) and Willer and Willer (2000) and other propositions are in the present paper systematically investigated using one framework; the effects will be assessed of adding and deleting a link on the payoff (i) of actors in the link (micro), (ii) of the neighbors of these actors (meso), and (iii) differences of all actors in the network (macro).

Our study differs in two important respects from the work of Leik (1992) and Willer and Willer (2000). Firstly, the two aforementioned studies make use of a specific theory of network exchange only to a limited extent. The propositions in those papers are too general to prove without choosing a specific theory of how actors exchange, that is, whether a proposition is true or not might depend on the theory of network exchange that is used to evaluate these propositions. Leik did not use any theory of exchange. Although Willer and Willer (2000) employed network exchange theory (NET), they did not prove all of their propositions and left some conjectures for future research (e.g., their Conjectures 1 (p. 261) and 3 (p. 268)). In the present study, we chose to use Friedkin's expected value theory as the theory for network exchange so that their propositions can be unambiguously evaluated. We motivate our selection of EVT in the next section, but will also argue later that at least some of our findings are robust across exchange theories. Secondly, in the evaluation of their propositions, Willer and Willer as well as Leik only refer to a limited number of exchange networks. In the present study all 13,597 connected and unconnected exchange networks with up to 8 actors are investigated. Examining all small networks has the advantage that one obtains an overview of how often a proposition is violated, if ever, and one might discover principles of dynamic exchange by inspecting networks in which propositions are violated.

In Section 2 the scope conditions underlying our analysis are discussed, our selection of Friedkin's (1992, 1993, 1995) expected value theory (EVT) is motivated, and EVT is discussed in detail. In Section 3 the propositions to be tested are explicated and related to propositions in Willer and Willer (2000) and Leik (1992). In Section 4, the results of our analyses are presented. We conclude with a discussion in Section 5.

2. Scope conditions and theory

2.1. Scope conditions

The scope conditions underlying our analysis are in line with those employed in the literature on exchange networks. These are the following (e.g., Leik, 1992; Markovsky et al., 1988; Willer and Willer, 2000):

(1) The number of nodes in the network is constant.
(2) Only directly linked nodes can engage in exchanges.
(3) All resource flows must be dyadic.
(4) The joint profit for any exchange is constant.

(5) All actors are involved in at most one exchange (the one-exchange rule). We additionally assume that
(6) All outcomes in exchange networks can accurately be described by a theory of exchange.

The last condition is similar to Condition (7) in Leik (1992) and Willer and Willer (2000) stating that 'Actors understand and use the principles of network power', but is more specific. Our aim here is to unambiguously evaluate the consequences of adding and deleting links, so that we need point predictions from one particular theory of network exchange. We selected EVT. The addition or deletion of a link in a particular network will not have the same effect in all exchange networks for one theory of exchange as for another.

2.2. *Selection of a theory of exchange*

Many theories of exchange networks have been developed and tested in the last three decades; power-dependence theory (e.g., Cook and Emerson, 1978; Cook and Yamagishi, 1992), exchange-resistance theory (e.g., Skvoretz and Willer, 1993), a graph-analytic theory using the graph-theoretic power index (GPI) (e.g., Markovsky et al., 1988), core theory (e.g., Bienenstock and Bonachich, 1992), optimal seek theory (Willer and Simpson, 1999), identity theory (Burke, 1997), Yamaguchi's (1996, 2000) rational choice model, expected value theory (e.g., Friedkin, 1992), and non-cooperative bargaining models (Berg and Panther, 1998; Braun and Gautschi, 2006). The collection of theories, exchange-resistance, GPI, and optimal seek, is sometimes abbreviated as network exchange theory (NET) by its developers.

We require the theory employed in our investigation to accurately predict resource pool divisions in laboratory experiments, to be well known in the field, and to yield unique point predictions for each exchange network. With respect to the first criterion, it seems that all the theories mentioned predict the outcomes of the exchange networks that are realized in the lab with reasonable accuracy. Four theories have received much more attention in the literature than the other theories. These are the approaches that have appeared in special issue 14 (3–4) of the journal Social Networks on exchange networks; core-theory, power-dependence theory, NET, and expected value theory.

Core theory is not a suitable method in our investigation because it does not provide unique predictions. For example, consider the Box network (Fig. 1). Core theory predicts that any two neighbors earn together precisely 24. Many outcomes satisfy this prediction, namely ($A = x$, $B = 24 - x$, $C = x$, $D = 24 - x$, $x \in [0, 24]$). In these solutions A and C obtain the same outcome x, and B and D both get $24 - x$.

Power-dependence theory often also does not provide unique predictions, and therefore does also not qualify for our investigation. Power-dependence theory is based on the equidependence principle. The consequence of the principle is that the difference between obtained payoff and conflict payoff is equal for both actors in each exchange relation that is carried out. The principle leads to non-unique predictions in many networks. For example, all core outcomes for the Box are equidependent.

The majority of studies on exchange networks are based on NET. It is somewhat difficult to describe NET because there are several different versions of the theory. Willer (1999) describes the graph-theoretic power index (GPI), and adapted versions of it like GPI-R or GPI-l_i, GPI-RD, GPI-l_i^2, and Iterative GPI. These adaptations of GPI combine GPI with the principle of exchange-resistance (Skvoretz and Willer, 1993; Van Assen, 2003), and/or algorithms to calculate probabilities that exchange relations are actually being carried out. These algorithms are called exchange seek likelihood (ESL) and optimal seek (OS). However, point predictions of exchange outcomes are not yet obtainable with OS. Willer and Willer (2000) use OS in their study on dynamic exchange networks, but never specify the OS algorithm. Girard and Borch (2003) have developed optimal seek simplified, which works out these unspecified details of OS. Emanuelson (2005) showed that GPI-R combined with optimal seek simplified outperforms the other variants of NET. However, no computer program yet exists of this variant, and because of its recency it has not yet received much attention in the literature on exchange networks. Consequently, we did not select NET to investigate the evolution of exchange networks.

We selected the remaining theory, Friedkin's (1992, 1993, 1995) expected value theory (EVT). Although uniqueness and existence is not proven for all networks, for every exchange relation in every network up to eight actors, it generates a unique point prediction and probability of exchange.

2.3. *Expected value theory (EVT)*

Building upon the theory of social power proposed by French (1956), Friedkin (1986) first suggested the idea of using expected values to predict the outcomes in a power structure. Friedkin (1992, 1993) extended the idea of expected values to analyze outcomes in an exchange network. Friedkin's model predicts the probability that each maximal exchange pattern occurs, and the outcome distribution in each of these patterns. A pattern is maximal in the sense that no further feasible transaction exists between actors that do not yet exchange. For example, the Line4 that has two maximal exchange patterns: $\{\{A-B\}, \{C-D\}\}$, and $\{\{B-C\}\}$. Using an iterative algorithm, each actor's expected payoff is calculated as the expected value of his payoffs over all possible maximal exchange patterns.

As opposed to what the name of Friedkin's theory suggests, EVT is not a theory based upon actors rationally maximizing their payoffs. The algorithm generating the predictions assumes that both actors' claim of their share of the 24 points in their relation is increasing non-linearly in the probability that each of them is excluded in any exchange. Three rules determine the final allocation in the relation. Which rule is applied depends on the sum of both actors' claims and their claims relative to half of the resource pool (12 points). An inconvenience of the EVT model is that because of the non-linear function and the three rules the algorithm is analytically intractable. See Friedkin (1995) for details of the EVT model that is used to generate the predictions.

3. Approach and propositions

The resource divisions for each exchange network are predicted with EVT (Friedkin, 1995). For all exchange relations in each network we check if they are candidates for deletion by comparing the EVT predictions before deletion of the link to the EVT predictions after deletion. Similarly, for all pairs of actors without a direct link we check if the absent links are candidates for addition. For adding a link between two actors mutual consent is required, while deletion of a link is unilateral. This makes perfect sense in exchange networks because exchange itself requires mutual consent.

We analyze a subset of all possible exchange networks; all 13,597 exchange networks of size 2 through 8 are investigated. Of these networks 12,112 are connected and 1485 are not (see sequence A001349 of Sloane's Online Encyclopedia of Integer Numbers). A network is connected if a path exists from each node to every other node. We need to consider unconnected networks as well, because through link deletion and addition networks can become unconnected and connected, respectively.

The propositions we test can be classified into three categories that differ with respect to the level of the network focused on. The propositions concern the effects of deleting or adding a link on (A) the payoffs of the actors that the link connects (micro-level), (B) the payoffs of the neighbors of these actors (meso-level) and (C) the overall payoff differential in the network (macro-level).

Proposition A1. *Adding a link between two actors increases the payoff of at least one actor and does not decrease the payoff of both actors.*

Proposition A1 is similar to the monotonicity axiom of rational actor behavior in a bargaining context. The monotonicity axiom states that if, for every payoff that an actor may demand, the maximum feasible payoff that one other actor can simultaneously reach is increased, then the payoff assigned to this other actor should also be increased (Kalai and Smorodinsky, 1975, p. 515). Proposition A1 also seems to be in line with EVT. An additional link decreases the probability of exclusion of the actors it connects. It gives them an additional alternative exchange partner and thus additional bargaining power through a decrease in the probability of exclusion as predicted by EVT.

If A1 does not hold, we would like to know why and when it does not hold. Leik (1992) and Willer and Willer (2000, p. 261 and 266) put forward in some of their propositions that in particular low power actors (actors that earn less or much less than actors in a network where everyone earns the same) want to add links, because they can gain most. We translate the line of thought that the more an actor earns, the less he can earn by adding a link, by formulating four specific versions of A1, depending on the individual and joint payoffs of the actors that form the link. For these propositions we need definitions of the average payoff of an actor in the network, the 'actor's share', and of the average sum of actor payoffs in a link, the 'link's share', in the network. Two definitions are employed for each share. One definition, share complete, is concerned with the average expected payoff of an actor and a link if the network were complete. If the network is complete, a maximal number of $N \operatorname{div} 2$ exchanges can be completed. $N \operatorname{div} 2$ is the mathematical notation for how often we can take two different actors out of N; if N is even, $N \operatorname{div} 2 = N/2$, and if it is odd then $N \operatorname{div} 2 = (N-1)/2$. MX denotes this maximal number of exchanges in the literature (e.g., Willer, 1999). The actor's and link's share complete are then defined as $\mathrm{MX} \times 24/N$ and $\mathrm{MX} \times 48/N$, respectively. For example, if $N = 9$ the actor's share complete is $4 \times 24/9 = 10.67$, and the link's share is twice that number. The link's share can be interpreted as the maximum payoff each actor can reasonably expect if all actors use all $N \operatorname{div} 2$ exchange opportunities and all actors are treated equally.

The share complete definitions do not take into account possible exclusion and non-connectedness in the current exchange network. Therefore we also define share current, which concerns the average expected payoff of actors in the current network. To calculate an actor's share current first the maximum number of exchanges possible given the network structure, denoted by NX in the literature (e.g., Willer, 1999), in i's sub-network needs to be calculated. If N_i and NX_i denote the size and NX of i's sub-network, then i's actor's and link's share current are defined as $\mathrm{NX}_i \times 24/N_i$ and $\mathrm{NX}_i \times 48/N_i$, respectively. For example, consider the nine-actor network that consists of two disconnected sub-networks, one connected pair and one 2S5B (two sellers, five buyers) network. Then the actor's share current in the pair and the 2S5B network is 12 and $2 \times 24/7 = 6.86$, respectively, and the link's shares are twice that number. The link's share can be interpreted as the maximum payoff each actor can reasonably expect, if all actors use all NX exchange opportunities and are treated equally. Note that the actor's share complete is 10.67 for each actor in the example.

We specify Proposition A1 for four different groups of actors, namely if initially,

(A1a) both actors earn more than their actor's share;
(A1b) one of the actors earns more than his actor's share, and their joint payoff exceeds the link's share;
(A1c) one of the actors earns more than his share, and their joint payoff does not exceed the link's share;
(A1d) both actors earn not more than their actor's share (Willer and Willer, 2000, p. 261).

In cases A1a and A1b both actors together already earn more than the link's share, therefore the added link might not increase their payoff. However, in case A1d both actors would definitely increase their payoff if they would only exchange with each other. Hence, if A1 is violated, it is most likely to be violated in cases A1a and A1b. With respect to propositions A1a to A1d, Willer and Willer suggest and believe to be true (but do not prove) that actors obtaining less than their share will benefit when they obtain more links to others (Willer and Willer, 2000, p. 270). Note, however, that their statement is not identical to any of the A1 propositions; our propositions are more specific.

Propositions parallel to A1a–A1d concern the effect of deleting a link on an actor's payoff.

Proposition A2. *Deleting a link between two actors decreases at least one actor's payoff and does not increase the other actor's payoff if initially,*

(A2a) *both actors earn more than their actor's share;*
(A2b) *one of the actors earns more than his actor's share, and their joint payoff exceeds the link's share;*
(A2c) *one of the actors earn more than his share, and their joint payoff does not exceed the link's share;*
(A2d) *both actors earn not more than their actor's share.*

The intuition and rationale behind A2 is similar to the one behind A1; by decreasing one's own number of links, one increases, or in any case does not decrease, one's probability of exclusion. Willer and Willer (2000, pp. 267–268) claim that 'no position in any network will prefer to delete any of its relations' (Willer and Willer, 2000, p. 268, Conjecture 2), implying that A2 is true, but do not prove it. We would argue that if A2 is violated, it is most likely violated in case A2a (they might not need a link) and least likely violated in case A2d (they need the link, the actors do not earn much yet).

The second set of propositions concern the effects of adding and deleting a link on neighbors' payoffs. An actor is called a neighbor if he has a link to at least one of the actors that add or delete the link. It is important to note that in our analyses the effects are analyzed of those additions and deletions that improve the payoffs of the actors that carry out these acts. Consequently, if A2 holds, a proposition concerning the effect of deleting a link on the neighbors' payoffs is no longer useful because no actor wants to delete a link. Our propositions on effects of the neighbor's payoffs are:

(B1) If an actor deletes a link, the payoffs of his neighbors do not decrease.
(B2) If an actor adds a link, the payoffs of his neighbors do not increase.

The intuition and rationale behind B1 and B2 is again the monotonicity axiom; adding (deleting) a link might increase (decrease) the payoffs or bargaining positions of the two actors in the link, thereby decreasing the payoff of the neighbor in his relation(s) with the actor(s) in the link. The original GPI theory of network exchange is also based on this logic (Markovsky et al., 1988).

Finally, many propositions of Willer and Willer (2000) are concerned with the effects of adding and deleting links on the overall power differential in the exchange network. However, they do not indicate how to measure 'overall power differential'. Here, we suggest to use the variance of all actors' payoffs as a measure of overall power differential. Some of their propositions can then be translated into:

(C1) Adding links does not increase the variance of payoffs (Willer and Willer, 2000, p. 253).

(C2) Deleting links does not decrease the variance of payoffs.

Leik (1992) and Willer and Willer (2000) relate overall power differentials to NX and MX (Leik, 1992, pp. 314–316; Willer and Willer, 2000, pp. 262–266). Willer and Willer disprove two of the three propositions, hence we will not deal with them. However, they argue that the following proposition, here C3, is true:

(C3) If NX < MX, then adding any link which increases NX will decrease overall power differential in the network (Leik, 1992, p. 314; Willer and Willer, 2000, p. 262),

Proposition C3 will be disproven with one counterexample, all other propositions will be examined by scrutinizing all networks up to size 8.

4. Results

Proposition A1 states that adding a costless link between two actors increases the payoff of at least one actor and does not decrease the payoff of both actors. Four versions of A1 were distinguished depending on how much each actor of the pair earns and how much they earn together compared to the 'actor's share' and the 'link's share'. The number of absent links across all exchange networks were counted and categorized into these four categories, and we checked if a pair of actors prefers the link to be added or not.

The second and fourth row of Table 1 show the total numbers of absent links in each category for each of the two definitions of share. The first and third row show the frequencies and percentages of these absent links for which the proposition holds. As can be expected, the frequency of absent links is related to the payoff of the actors in the pair; most absent links are found in pairs that do not earn more than their share (columns A1c and A1d), while there are fewer absent links in pairs that earn more than their share (columns A1a and A1b). Additionally, the number of absent links

Table 1
Frequencies (percentages) of links added, depending on the individual actor's payoff and a pair's joint payoff compared to the actor's and link's share

	A1a	A1b	A1c	A1d
Added 'com'	7,229 (54.71)	9,459 (55.31)	35,968 (97.06)	117,430 (99.68)
Total 'com'	13,214	17,101	37,059	117,812
Added 'cur'	7,780 (54.01)	15,787 (66.46)	34,258 (99.66)	112,261 (99.65)
Total 'cur'	14,014	23,753	34,373	112,655

Share was computed either for the current network ('[share] cur[rent]') or for a complete network ('[share] com[plete]').

depends on the definition of share we consider. Since 'share complete' is seldom less than 'share current', more absent links are categorized in A1c and A1d using the 'share complete' definition. The first number 7229 (54.71) in the first row implies that there are 7229 (out of in total 13,214, or 54.71%) absent links for actors that both earn more than their 'share complete' that the two actors do not want to add.

Table 1 clearly demonstrates that A1 has to be rejected. Pairs of actors do not always prefer to connect. In particular when their joint payoff is higher than the link's share, the pair often does not want to add the link under both share definitions (for categories A1a and A1b, respectively, 45.29% and 44.69% [share complete] and 45.99% and 33.54% [share current]). One explanation for this finding is the following. Friedkin's method predicts that the added link will be used with a positive probability. However, for many links of categories A1c and A1d this link is suboptimal for at least one of the two actors in the link; he can earn more in another exchange relation. Hence the use of an added link will likely results in a decrease of the average payoff of at least one of the actors. A simple example is the five-actor exchange network consisting of a Line3 (Fig. 1) and a connected pair. The expected payoff of actor *B* in the Line3 is 21.1 points, and 12 for both actors in the pair. When *B* connects to one of the actors in the pair, thereby creating the so-called T-network, *B*'s expected payoff decreases to 20.70 because the expected probability that this link is used is 0.07.

A1d is concerned with pairs of actors where none of the two earns more than his share. It might be the case that the small percentage of pairs who do not wish to add the link (0.32% and 0.35%) contains only pairs of actors earning exactly their share. Therefore, we also checked whether there are pairs of actors where both actors earn less than their share and still do not want to establish a link between them. Such a pair was found in 39 (0.3%) of the exchange networks when defining share for the complete network, and no pair was found when defining share for the current network. No violation of A1d in all networks up to size 8 was found using current network share. In any case, the results indicate that, almost always, actors having no link establish a link when both of them earn less than their share in the network.

An exchange network where A1d is violated using share complete is depicted in Fig. 2. The depicted eight-actor exchange network is special because it can be used to show that all propositions are false if complete network share is used. In the figure, three types of numbers are shown. The actors' expected payoffs are represented by numbers in bold, and the expected probabilities

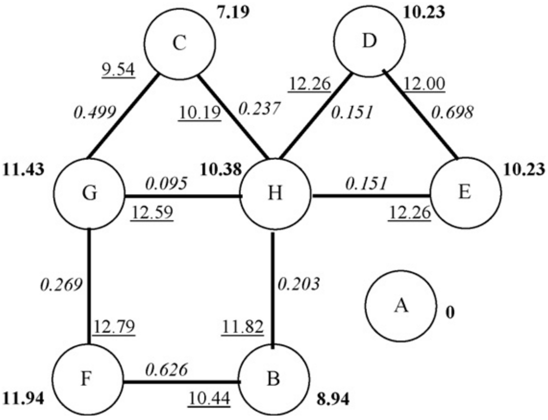

Fig. 2. Example of an exchange network in which all propositions are violated using share complete.

Table 2
Frequencies (percentages) of links deleted, depending on the individual actor's payoff and a pair's joint payoff compared to the actor's and link's share

	A2a	A2b	A2c	A2d
Deleted 'com'	6,055 (29.65)	8,436 (23.36)	1102 (1.43)	55 (0.11)
Total 'com'	20,420	36,115	77,305	51,346
Deleted 'cur'	6,780 (29.82)	8,858 (19.53)	8 (0.01)	2 (0.00)
Total 'cur'	22,738	45,364	70,265	46,819

Share was computed either on the current network ('[share] cur[rent]') or on the assumption of a complete network ('[share] com[plete]').

of using an exchange relation in italics. For each exchange relation one underlined number is shown; this is the expected payoff of the first actor (in the sense of alphabetical order) in the relation when this exchange is carried out. There is (only) one pair in the exchange network that does not want to connect, although their joint payoff is smaller than 24. This is the pair FH. *F* does not want to add the link because his payoff then decreases from 11.94 to 11.81. *H* would benefit from the addition, his payoff would increase from 10.38 to 10.46. The main cause of the decrease of *F*s expected payoff is that, because of the addition, the probabilities that *F* exchanges with *B* and *G*, who were his most profitable exchange relations, decrease.

A2 states that deleting a link between two actors decreases the payoff of at least one actor and does not increase the payoff of an actor. Consequently, it is expected that actors never delete one of their links, independent of the category to which the actor's link belongs. Table 2 shows the number of existing links and how often an actor wants its deletion, separately for both share definitions and using categories A2a–A2d, similar to the categories for A1. Table 2 has a similar structure as Table 1.

A2 is rejected because in more than 20% of the links where the joint payoff is higher than the link's share, at least one of the actors wants it deleted. The explanation is similar to the explanation of the falsification of A1; the deletion of the link might increase at least one actor's payoff because its (infrequent) use leads to a decrease in expected payoff.

If the joint payoff is less than the link's share, the link is only infrequently deleted (for categories A2c and A2d, respectively, 1.43% and 0.01% [share complete] and 0.11% and 0.005% [share current]) because by making use of that link the actors can get more than what they get in their expected payoff. We also checked if there were pairs where both actors earn less than their share and at least one actor wants to delete the link. Such a pair was found in 40 of the exchange networks using complete share, and in two networks using current share.

Consider Fig. 2 for an example of a violation of A2d using complete share. In this exchange relation both *G* and *H* want to delete their link although together they earn less than 24 points. After deleting the link, *G*s and *H*s expected payoff increases from 11.43 to 11.64 and from 10.38 to 10.39, respectively. Because of the deletion, *G*s and *H*s expected payoffs in their exchanges with *C* decrease, but this negative effect is more than compensated for by the fact that for both *G* and *H* their relation was less profitable than the other relations.

Propositions B1 and B2 concern the effect of actual additions and deletions of links on payoffs of neighbors instead of payoffs of the actors themselves. It was hypothesized that a neighbor's payoff cannot increase after an action (deletion or addition) that was carried out to improve the payoffs of the actors in the link. To investigate the effect of such an action the changes in payoff of all actors neighboring the actors in the link were computed, using both share definitions. B1 states that if an actor wants to delete a link, his neighbors will not profit from it. However, of all

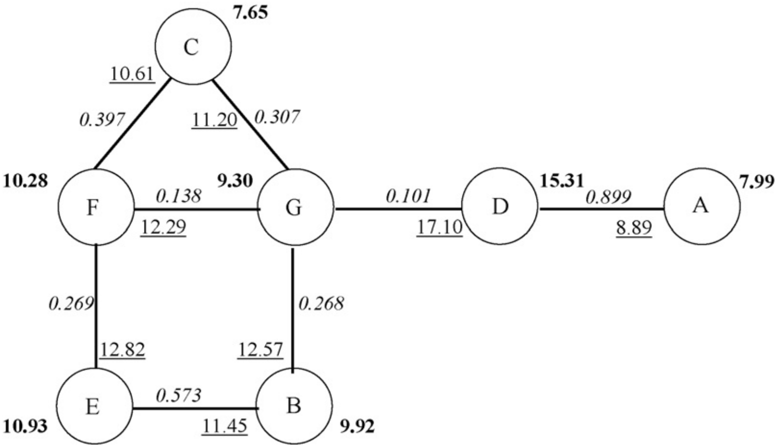

Fig. 3. Example of an exchange network in which all propositions except A1d are violated, using current share.

neighbors of actors deleting a link a substantial proportion actually profit from this deletion; of in total 57,199 neighbors of actors deleting a link, 34,185 (59.77%) profited from the deletion of the link. For example, as explained earlier, *C*s expected payoff increases after the deletion of the link between *G* and *H* in Fig. 2. It increases from 7.19 to 8.16.

In Fig. 3 an example is provided of a network where A2d is violated using share current. The notation in Fig. 3 is identical to that used in Fig. 2. Note that the seven-actor network in Fig. 3 has a structure similar to the eight-actor network in Fig. 2. In this network, the actor's (current) share is $3 \times 24/7 = 10.286$. Both *F* and *G* earn less than their share, *F* only just slightly. However, *F*s expected payoff increases to 10.29 after deleting the link with *G*, while *G*s payoff decreases to 9.06. This example suggests that also A1d might be violated in exchange networks of size larger than 8, when in a network the expected payoffs of both actors without having a link is just below their share current.

Additionally, B2 is falsified; additions of a link in some cases have a positive effect on the neighbor's expected payoff. Of 726,423 neighbors of added links, the expected payoff increased for 146,678 actors (20.2%). In our exchange networks in Figs. 2 and 3, one or more neighbors' payoffs increase in 17 and 9 possible link additions, respectively. For example, *H*s expected payoff increases from 10.38 to 10.75 in Fig. 2 when the link between *A* and *B* is added.

The results concerning the effect of a deletion or addition of a link on the overall payoff differential, as measured with payoff variance, led again to a falsification of our propositions. In 5421 (39.87%) of the exchange networks a deletion of a link resulted in an increase in the payoff variance. This is a large number when we take into account that in many networks no actor wants to delete a link. Moreover, in 12,743 (93.72%) of the networks the addition of a link resulted in an increase of the payoff variance. In the Fig. 2 network, the variance of payoffs is increased both after deleting the link between *G* and *H*, and after 16 possible link additions, e.g., a link from A to another actor.

C3 states that if $\text{NX} < \text{MX}$, then adding any link which increases NX will decrease payoff variance. One counterexample is presented to reject C3. Consider a six-actor exchange network consisting of two unconnected triangles. In this network only two exchanges can be carried out. Each actor's expected payoff in the network is 8 and payoff variance is 0. Each actor prefers to add a link to an actor of another triangle, resulting in an increase of NX with 1, i.e., then three exchanges can be completed. After connecting the two triangles the expected payoffs of the actors joining the two triangles increase to 9.55, while the other four actors' expected payoffs increase to 8.97. Note that the payoff variance is larger than 0.

5. Conclusions and discussion

Network change is most likely in contexts in which a slight local perturbation can alter the payoff distribution over the people in that locality significantly. The context of network exchange fits this description perfectly. Yet, while decades of extensive laboratory research have advanced the prediction of payoff distributions in exchange networks to the stage of fine-tuning, the implications of perturbing the network structure for the actors' payoffs have hardly been investigated.

We have examined the validity of propositions concerning the effect of adding and deleting a link on (A) the payoffs of the actors that the link connects (local or micro-level), (B) the payoffs of a neighbor of these actors (meso-level) and (C) the variance of payoffs in the network (global or macro-level). To derive effects of addition and deletion we selected Friedkin's EVT. We investigated all networks of size 2 through 8, such that an overview was obtained how often and in which networks a proposition was violated.

All propositions, some of which were taken from the literature, were falsified. The proportion of counterexamples to some was so small that some propositions could be established as general rules. In general, adding (deleting) a link increases (decreases) the gains of the actors involved if before the addition (deletion) they together earned in total not more (less) than their link's share. However, no general rule holds concerning the effect of adding or deleting a link on higher levels of network outcomes, i.e., neighbors' payoffs and the variance of payoffs in the network.

The finding that adding (deleting) a costless link can decrease (increase) an actor's payoff may be puzzling to some. Indeed, this finding violates the intuitively plausible rationality condition of monotonicity. From a rational choice point of view on exchange networks, if a link is added one can always decide not to use it, so one's payoff should not be negatively affected. However, EVT is not a rational choice model. It predicts that people also use exchange relations that are sub-optimal, e.g., the link connecting the Line3 and the pair in the T network. Other theories of exchange will also predict violations of the monotonicity axiom, if they predict irrational behavior like using sub-optimal links. Both PD theory and NET also predict irrational behavior. For example, although PD theory is silent on the probability that an exchange relation is used, a sub-optimal exchange relation is predicted to be used 'not very often' (Cook and Yamagishi, 1992, p. 252). Also NET assumes that sub-optimal relations are used, as long as they are not breaks, i.e., links that are never used. Consequently, our results concerning violations of monotonicity are robust to selection of the theory, if the theory assumes that sub-optimal relations are used. That is, it might well be that while applying PD, NET or another theory that violates the monotonicity condition, the respective propositions are falsified as well. This is an issue for future research.

Another extension for future research is to study the evolution of exchange networks as pairs of actors add profitable links and delete harmful ones, and to identify stable networks to which no actor wishes to make any further change. Recently a methodology has been developed to perform such investigations (e.g., Dutta and Jackson, 2003). Due to the aforementioned theoretical and empirical untenability of stasis, exchange networks are a perfect application.

Last, some first experimental explorations of network dynamics were recently conducted (cf. Berninghaus et al., 2005). These experiments investigate how actors decide what links to add and delete given exogenously determined incentives. A first experimental investigation of exchange network evolution, however, is still to be conducted.

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