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Theories of network exchange: Anomalies, desirable properties, and critical networks

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ABSTRACT

The present study evaluates four well-known theories of exchange in networks. In previous research these theories' predictions were compared for a small set of networks using experimental data. We compared their predictions for all 12,112 networks up to size 8. By comparing these predictions we (i) identified anomalies in theories of network exchange, (ii) investigated to what extent the theories satisfy basic principles of exchange, (iii) identified 'critical' networks for which predictions are very different. We conclude that exchange in networks is not yet well understood.

1. Introduction

An exchange situation can broadly be defined as a situation involving people who have the opportunity to collaborate for the benefits of everyone involved. While exchange has been intensively studied in economics for more than a century since Edgeworth (1881), exchange entered the fields of social psychology and sociology only in the second half of the twentieth century. Homans (1958, p. 606) introduced the idea that 'social behavior is an exchange of goods, not only material goods but also non-material ones, such as the symbols of approval and prestige'. Since the seminal studies of Stolte and Emerson (1977) and Cook and Emerson's (1978), sociologists have focused on the effect of social structures on outcomes of exchange. The basic notion of this research is that social behavior is shaped by the social relations in which it occurs, which are in return conditioned by the structure or 'exchange network' within which they are embedded (Willer, 1999, p. xiii). The exchange network represents opportunities and restrictions to exchange. These opportunities and restrictions to exchange arise naturally in many real-life situations. Two of the most common causes for the absence of an exchange relation between two persons are natural barriers and non-matching preferences. Examples of barriers are not knowing each other, or not being able to contact or meet each other. And two persons might also not have an exchange relation because one of them has nothing to offer that is valuable enough to the other, like sellers of the same product in a so-called buyer–seller (exchange) network.

The sociological conceptualization of "network exchange" is thus general in scope. It includes any kind of multi-person mutually advantageous endeavor in which barriers to interaction somehow condition behavior. The scope of research on network exchange, however, has remained rather limited (e.g., Cook and Emerson, 1978; special issue Social Networks June 1992; special issue Rationality and Society January 1997; Molm, 1997; Willer, 1999). With the exception of a few studies, it considers a rather specific exchange setting and investigates only a small and arbitrary collection of networks. The former restriction, the specificity of the exchange setting, has been discussed elsewhere (e.g., Braun and Gautschi, 2006, pp. 2–3). Our focus is on the latter restriction, the small and arbitrary collection of investigated exchange networks, and its possible consequences for research on network exchange.

The network exchange setting can be described as follows. Network structure determines which actors can exchange with whom, where a tie signifies a bilateral exchange relation. An exchange relation usually constitutes, as in the present paper, the opportunity to split a common resource pool of 24 points. Successful exchange occurs if two connected actors can agree on a division. There is an upper limit to the number of exchanges that an actor can engage in. Usually this limit is 1 (the so-called "one-exchange rule"), as also in this paper.

Several theories have been proposed that generate predictions of resource divisions in this exchange setting. These theories more or less agree on the effects of network structure in a few well-studied networks. In some networks, actors obtain equally profitable exchange outcomes (Bonacich and Bienenstock, 1995; Bonacich, 1998), for example, all actors in the Full4 (Fig. 1a). Therefore, such networks have been called 'equal power'. Other networks consist of actors who have a clear advantage and obtain most of the

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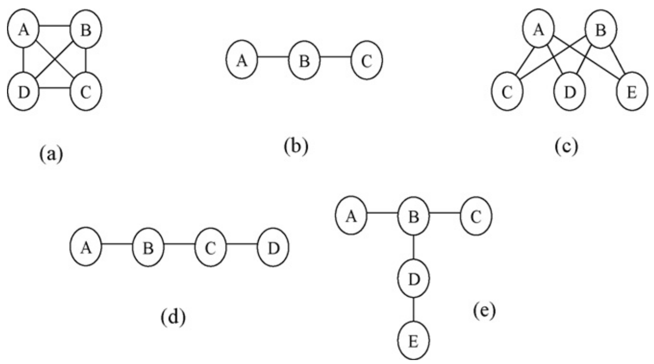

Fig. 1. Some exchange networks. (a) Full4. (b) Line3. (c) 2S3B. (d) Line4. (e) T.

profit in their relation as opposed to other actors who gain almost nothing. E.g., B in the Line3 (Fig. 1b) obtains most of the profit in his exchange with either A or C (e.g., Willer, 1999), and A and B obtain most of the profit in their exchanges in the 2S3B network[1] depicted in Fig. 1c (Willer and Willer, 2000; Corominas-Bosch, 2004). These networks are called 'strong power'. In yet other networks profits differ only slightly across actors. E.g., B (C) in the Line4 (Fig. 1d) obtains a marginally larger share than A (D) in their exchange relation (e.g., Willer, 1999). These networks are called 'weak power'. Finally, some networks are combinations of equal-, strong-, and weak-power parts. For example, part ABC of the T network (Fig. 1e) is strong power while DE is equal power according to some theories and weak power according to other theories.

Experiments, carried out on approximately 30 exchange networks (Van Assen, 2003), revealed that in general the theories' predictions of resource divisions more or less agree with each other on these networks. Willer (1999, p. 298) concludes that "The conditions of power in strong power structures are well known, interactions in strong power structures are well understood, and strong power structures can easily be produced in the lab." It is also claimed that theories of exchange are internally consistent, e.g., Emanuelson and Willer (2007, p. 26) state that the most recent version of one of the theories, Network Exchange Theory, is "free of contradictions". However, as we will demonstrate, theories disagree on the conditions of power in exchange networks, and Network Exchange Theory and other well-known theories make anomalous predictions for many networks that were never analyzed that contradict the very theories from which they were derived.

Van Assen (2003, p. 90) demonstrated that the subset of networks investigated are unrepresentative; they on average have a low density, and the incidence of strong power networks is much larger than in the whole population of networks of small size. Nevertheless, theories of exchange are fine-tuned on this small and unrepresentative set of networks. A typical evaluation of competing theories involves the computation of the fit across a handful of empirically investigated networks. The most recent and most ambitious evaluation of this type is Emanuelson and Willer (2007) who compare ten exchange theories for eleven exchange relations in seven networks.

The *empirical* confinement to a small number of networks is unavoidable. One may perhaps pick the few networks one wishes to investigate more strategically, but ultimately one can only study so many networks in the laboratory. The *theoretical* confinement to a small number of networks is *not* unavoidable. By broadening the theoretical scope to include more networks, we hope to make three contributions. First, we point out anomalies in exchange theories that previously uninvestigated networks bring to light.

Second, we perform an axiomatic analysis of exchange theories. More specifically, we investigate whether network exchange theories satisfy, and if not, how often they violate, six basic axioms or desirable properties. The aim of the axiomatic analysis is to increase our understanding of theories of network exchange. Until recently research merely focused on comparing point predictions of theories for a small set of networks, and not on regularities of a theory's predictions for any network. Focusing on these regularities will inform us on what the implications are of assuming one theory of exchange instead of another. The regularities of theories' predictions can be inferred by examining whether these theories satisfy properties. The six properties we examine can be distinguished into two classes. First, we improve our understanding of theories by examining whether they always specify a solution (existence), and if they do, if they specify one or more solutions (uniqueness). The second class contains four properties that are axioms in the literature on cooperative bargaining. By focusing on these four properties we can establish to what extent sociological exchange theories differ from economic exchange theories. Moreover, as we will argue in Section 3, each of the four axioms we selected from the cooperative bargaining literature translates into a corresponding principle that has been central in the sociological exchange literature. As a result, our axiomatic analysis also allows us to determine how well each of the exchange theories incorporates these central sociological principles of exchange.

The third contribution is that we identify networks for which the predictions of network exchange theories differ maximally. Such networks will more easily differentiate the theories in empirical fit and are therefore more cost-effective in data collection. The existence of such networks with large differences in prediction reveals that theories disagree on the conditions of power in exchange networks.

The following four well-known theories are analyzed: Core Theory (Bienenstock and Bonacich, 1992; Bonacich and Bienenstock, 1995; Bonacich, 1998), Power–Dependence Theory (Cook et al., 1983; Yamagishi et al., 1988; Cook and Yamagishi, 1992), Network Exchange Theory (Willer, 1999), and Expected Value Theory (Friedkin, 1992, 1993, 1995). We selected these theories because no theories have been investigated as much and received as much attention as these theories (e.g., see the special issues of Social Networks June 1992 and Rationality and Society January 1997). We computed predictions for these four theories for all 12,112 connected networks of size 2–8. This allows us to analyze for the first time their predicted exchange outcomes for all of the 170,884 relations in the 99.9% of small networks that have never been studied either theoretically or empirically.

One quantitative study of network exchange preceded ours. Van Assen (2003) analyzed and compared the performance of Power–Dependence Theory (PDT) and Network Exchange Theory (NET) on all networks up to size 9. However, his implementation of these two theories is different from ours, and hence also his results. First, he did not implement PDT as described by Cook and Yamagishi (1992), who developed the algorithm to generate predictions for PDT. Second, he generated predictions using only the equiresistance principle of NET. We use the most recent version of NET advocated by its developers (Emanuelson, 2005; Emanuelson and Willer, 2007). And lastly, he did not carry out an axiomatic analysis.

In Section 2 we review the four theories and point out anomalies in exchange theories. In Section 3 we discuss the axioms and perform the axiomatic analysis of the four theories. In Section 4

[1] The 2S3B network can be considered a B(uyer)–S(eller) network; either seller sells his one and only product to one of three buyers, each buyer wants to buy only one product, and the surplus (24) of the purchase is independent of which buyer buys from which seller.

we identify networks for which the predictions of the four theories differ maximally from one another. The paper concludes with a discussion section.

2. Theories and anomalies

We review the four theories in four separate subsections. At the end of each subsection, we point out anomalies in the theory and exemplify these anomalies with predictions for previously uninvestigated networks.

2.1. Core Theory (COT)

The concept of "core" as applied in network exchange stems from cooperative game theory with transferable utility (Shapley, 1952; Gillies, 1953). The core is the set of all exchange outcomes that satisfy three conditions; individual, coalitional, and collective rationality. *Individual rationality* requires that each individual earns at least as much as she could secure herself alone. *Coalitional rationality* means that each coalition should together earn at least what they could guarantee themselves without cooperation of those outside the coalition. *Collective rationality* demands that all individuals should together earn maximal profit. The application of the core to network exchange is referred to as "Core Theory" (COT).

The division of profit in an exchange network is not a cooperative game with transferable utility. This seems to make the application of the concept of the core difficult for two reasons. First, contracts are decided on in pairs, not network-wide. And second, coalitions cannot freely distribute their payoffs across their members, i.e., 'transferable utility' does not hold. For example, the payoffs of the coalition ABCD in the Line4 network (Fig. 1d) are 48, but A and B together earning more than 24 is infeasible. Bienenstock and Bonacich (1992) nevertheless apply the concept of the core to exchange networks. Bonacich and Bienenstock (1995) defend their approach by showing that if in the network exchange game ". . . coalitional rationality holds for every dyad, it will also hold for every larger coalition if the one-exchange rule is in effect." (p. 314). In other words, if actors exchange such that all pairs of connected actors obtain at least the value of their two-member coalition, i.e., if dyadic rationality or outcome stability holds, then the resulting allocation is in the core. This fixes the problem of contracts in coalitions with more than two members; since rationality of connected dyads guarantees rationality in all larger coalitions, it is not necessary to assume the possibility of contracts in larger coalitions. The rationality conditions on allocations to individuals and connected dyads also exclude infeasible allocations from the core, which solves the second problem of limits to the transferability of utility.

Bonacich (1998, p. 190) also gave a behavioral justification of the core. He showed that if actors in the exchange network bargain following four simple and plausible behavioral rules, then outcomes in the core are stable and outcomes outside the core are unstable. The four rules are: (1) Actors who are excluded from an exchange raise their offers in the next game, (2) actors who are included in an exchange make the same offers to others, (3) all offers made by actors to others are equal in value (to themselves), and (4) an actor leaves a current partner if and only if she receives a strictly superior offer from another.

Bonacich (1998) organizes the three rationality conditions in a linear system, which he solves to obtain the core. We also apply Bonacich's (1998) procedure for obtaining the core in the present paper. Bonacich distinguishes four types of core solutions. (1) Outcome stability does not hold and the core is empty; rational exchange cannot occur. (2) The core allocates an entire resource pool to some and zero to all others. (3) The core allocates (1/2) to all actors. (4) The number of solutions is infinite.

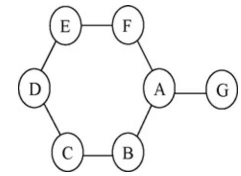

Fig. 2. A 7-actor network.

2.1.1. The problem of simultaneity in COT

COT requires rationality in all exchange relations simultaneously. However, actors cannot exchange simultaneously in the absence of central coordination. What is irrational in simultaneous exchange may be rational in sequential exchange. Consider the 7-actor network in Fig. 2 to illustrate the conflict between rationality in sequential and simultaneous exchange. COT predicts a profit vector of (24, 0, 24, 0, 24, 0, 0) for A through G as the uniquely rational distribution. Every relation appears in some maximal exchange pattern, so that any two connected actors could rationally exchange first. Suppose, however, that A and B exchange first. Then an isolate and a Line4 remain. COT applied to the whole network predicts C to obtain 24 points, but COT also predicts that C does not have an advantageous position in the remaining Line4. Since exchange in the lab is sequential, the theory is not consistent with the design of experiments on exchange, and false predictions might result from this inconsistency. For example, because exchange is sequential we predict using a rationality analysis that E and C will earn less than A in the 7-actor network. C and E can be expected to earn less than A because B or F might exchange with A instead of G.

As we will see next, PDT makes the same assumption of simultaneous exchange, leading to anomalous predictions.

2.2. Power–Dependence Theory (PDT)

PDTs concepts of Power and Dependence stem from early work on exchange by Emerson (1962, 1972a). In Emerson's (1962) original formulation of PDT only dyadic exchange was considered. In subsequent work, Emerson and colleagues (Cook and Emerson, 1978; Cook et al., 1983, 1986; Cook and Gillmore, 1984; Cook and Yamagishi, 1992; Emerson, 1972b, 1976; Stolte and Emerson, 1977; Yamagishi et al., 1988) extended the logic of PDT to apply also to exchange networks. Takahashi and Yamagishi (1993) adapted PDT by incorporating additional assumptions on the probability that a particular exchange is carried out. In the present study the predictions are derived with a more parsimonious version of PDT similar to the version described by Cook and Yamagishi (1992). Other studies that also compared predictions of different theories also used Cook and Yamagishi's (1992) version (Burke, 1997; Skvoretz and Willer, 1993; Braun and Gautschi, 2006; Buskens and van de Rijt, 2008; Emanuelson and Willer, 2007).

The logic of PDT is based on the idea that the two actors in an exchange relation compare their possible profits in the exchange to the possible profits they can obtain in an exchange with another actor, the so-called *conflict payoffs*, symbolized with C. PDT's claim is that the actors' profits in their exchange relation, relative to their conflict payoff, are equal. More formally, if actors j and k have the possibility to exchange then the exchange outcomes of j (O_j) and k (O_k) in relation jk are such that

$$O_j - C_j = O_k - C_k \quad (1)$$

The left hand of the equation is called the dependence of j on k, the right hand the dependence of k on j. The theory claims that the actors exchange at a ratio at which their dependence on each other is equal. The theory's claim is also known as the *equidependence principle*. If we assume that both actors in an exchange relation completely divide up the 24 points, then after substituting $O_k = O_j - 24$ we obtain

$$O_j = \frac{24 + C_j - C_k}{2} \tag{2}$$

Eq. (2) can be used to predict outcomes of exchange in case of positive dependencies between actors in (1).

The dependence of actors on each other can also be negative in some exchange relations in a network. For example, consider the BC relation in the *Line4*. PDT predicts that B (C) earns more in her relationship with A (D) than in her relationship with C (B). Consequently, the sum of their conflict payoffs, $C_B + C_C$, is larger than 24. Since B and C can get less from their relation than from their partners the exchange relation is called *suboptimal*. A suboptimal exchange relation is predicted to be used 'not very often' (Cook and Yamagishi, 1992, p. 252), but PDT is silent on the probability that the exchange relation is used.

Cook and Yamagishi (1992) do not use (1) to predict the exchange outcomes in a suboptimal exchange relation. They argue that, because the sum of conflict payoffs exceeds 24, the maximum profit an actor can obtain is limited by the other actor's conflict payoff; the other actor can always get a payoff at least as large as her conflict payoff, and will never accept less. Therefore, Cook and Yamagishi (1992, p. 252) use

$$O_j = 24 - C_k \tag{3}$$

to calculate payoffs in suboptimal relations. Note that $O_j + O_k < 24$ in these relations, because $C_k + C_j > 24$.

Cook and Yamagishi (1992) describe an algorithm using (1)–(3) for determining equidependent exchange ratios in any exchange network. This algorithm first assigns an equal split to each relation. Then it computes the current conflict points for all relations. Then it adjusts the predicted outcomes to these conflict points. Thereafter it computes new conflict points based on these new predictions, etc., until all predicted outcomes are equidependent. Applying the algorithm to the *Line4* results in $O_A = 8$, $O_B = 16$ ($O_C = 8$, $O_D = 16$) in relation AB (CD), and $O_B = O_C = 8$ in relation BC.

Van Assen (2003) applied the algorithm of Cook and Yamagishi (1992) but only used (1), that is, he treated suboptimal exchange relations similar to the other relations. The consequence of using only (1) is that also in suboptimal exchange relations all 24 points are divided. For example, if the algorithm of Van Assen (2003) is applied to the Line4 then the outcomes are $O_A = 6$, $O_B = 18$ ($O_C = 6$, $O_D = 18$) in relation AB (CD), and $O_B = O_C = 12$ in relation BC.

In our analysis comparing the predictions of the four theories we attempt to stick to the original PDT as closely as possible. The algorithm of Cook and Yamagishi (1992) predicts no exchange patterns but generates predictions for exchange relations that are not suboptimal. The algorithm's outcomes of suboptimal relations cannot be used as predictions, since in the lab all 24 points are always divided. Predictions in these relations will be computed by (2) *after* the algorithm has converged. Applied to the Line4, we obtain the same predictions for the AB and CD relations as the original algorithm (16 for B and C) because these relations are not suboptimal, and 12 points for B and C in relation BC.

PDT of Cook and Yamagishi (1992) is silent on the probabilities of occurrence of exchange relations. Consequently, we can only use PDT for the prediction of divisions of resource pools in exchange relations but not for the computation of expected payoffs of actors in the network.

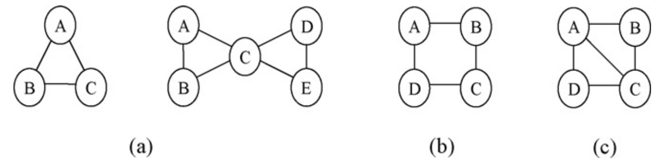

Fig. 3. Some exchange networks. (a) Triangle and Kite. (b) Box. (c) D-Box.

2.2.1. Anomalies in PDT

In many networks an infinite number of outcomes satisfy the PDT requirements. We demonstrate this result analytically for two different classes of networks. Consider a large class of exchange networks satisfying the following requirements when actors are of two types, yellow and blue[2]: (i) each actor has at least two links to actors of the other color, (ii) there are no links between same-colored actors. Call this network *nu-sym* (for non-unique symmetric). Note that a *nu-sym* network is a *bipartite* network (Wasserman and Faust, 1994, p. 120-1), but that not all bipartite networks are *nu-sym* because there are bipartite networks in which at least one actor has only one link.

Theorem 1. *If a network is* nu-sym, *then all solutions characterized by* $0 \le x \le 24$ *for the yellow actors, and* $24 - x$ *for the blue actors, are PDT solutions.*

Proof of Theorem 1. For each relation each actor has an alternative relation, because each actor has at least two ties in a *nu-sym* network. Consider any of the characterized solutions. An actor earns the same in each of his exchange relations. Dependence is therefore 0 for all actors in all relations. □

The algorithm of Cook and Yamagishi fails to identify all these solutions but specifies the solution in which all actors obtain 12 in their relations as *the* solution. In some networks, the 'all-12' outcome can be defended as *the* solution, using symmetry arguments (property SYM as defined in the next section). For example, consider the Box (Fig. 3b). Since all actors are in automorphically equivalent positions, there is no reason to expect that one actor obtains more.

However, in some networks the symmetry argument cannot be applied to defend the 'all-12' outcome in *nu-sym* networks. Consider the 2S3B network (Fig. 1c). The PDT algorithm allocates 12 to each actor in each relation, but the symmetry argument cannot be applied because the sellers and buyers are not in equivalent positions. An infinite number of alternative allocations with all buyers earning the same amount and all sellers earning the same amount also satisfy symmetry.

While no symmetry argument can be made to defend this prediction for the 2S3B network, there are good arguments to question the 'all-12' prediction. Economic theory predicts that the three buyers will outbid each other and at the end will obtain a very small outcome, and the sellers will obtain (almost) the entire surplus. The Core algorithm of Bonacich finds this as the unique solution, allocating 24 to the sellers and 0 to the buyers. But one does not need to revert to economic theory to consider these PDT predictions anomalous. Also Markovsky's simulation model (1987), assuming actors with very limited rationality, predicts a payoff vector with minimal earnings for the buyers.

PDT predicts that each seller in the 2S3B gets $24 \ge x \ge 0$, and each buyer $24 - x$: Even an allocation of zero profit to all sellers is equidependent. Hence the 2S3B demonstrates not only that Cook

[2] We choose colors yellow and blue and not red and green, because a larger portion of the human population is color blind for red and green than for blue and yellow.

and Yamagishi's algorithm fails to correctly specify the solution, but also that PDT itself fails for some networks. These networks are a subset of all bipartite networks, including all networks with multiple sellers all connected to each of multiple buyers with, an unequal number of sellers and buyers.

There is also a class of networks for which PDT generates an infinite number of solutions that are not symmetric, as pointed out by Theorem 2. Consider a large class of exchange networks satisfying the following requirements when actors are of two types, yellow and blue: (i) each actor has at least two links to an actor of a different color, (ii) there is at least one connection between yellow actors, but none between blue actors. Call this network *nu-asym*.

Theorem 2. *If a network is* nu-asym, *then all solutions characterized by* 12 *for yellow actors in yellow–yellow relations, and* $24 \geq x \geq 12$ *for yellow actors and* $24 - x$ *for blue actors in yellow–blue relations, are PDT solutions.*

Proof of Theorem 2. Consider a yellow–yellow relation. Both actors have an alternative yellow–blue relation in a nu-asym network in which they earn x. For any of the proposed solutions in which $x \geq 12$, both actors in this relation therefore have conflict payoff x. Their dependences are equal for a (12, 12) allocation, namely both $x - 12$. Consider a yellow–blue relation. The yellow actor in a nu-asym network has another relation with a blue actor yielding him a conflict payoff of x. The blue actor has one or more alternative relations with exclusively yellow actors in which he earns $24 - x$. The $(x, 24 - x)$ allocation gives both actors dependence $(24 - x) - (24 - x) = x - x = 0$. □

An example of a *nu-asym* network is the Box with one diagonal ("D-Box", Fig. 3c). Note that the algorithm of Cook and Yamagishi again specifies 'all-12' as the solution. However, one might argue that the actors on the diagonal can or should obtain more, since they have the power to exclude the off-diagonal actors. Also note that in a class of *nu-asym* networks the PDT prediction is clearly false. An example of such a class is the set of networks with multiple sellers all connected to each of multiple buyers with an unequal number of sellers and buyers, and with links between the actors of the group that is smaller in number. For example, the network obtained by connecting the sellers with one another in the 2S3B network.

These anomalous predictions can be traced back to the assumption of simultaneous equidependence in all relations. Buskens and van de Rijt (2008) replace this assumption by the assumption that equidependence should hold only at the moment of exchange, that is, they assume sequential instead of simultaneous exchange. Their adjustment of PDT leads to the prediction of a unique payoff vector with 24 for all sellers in the S23B network.

2.3. Expected Value Theory (EVT)

Friedkin (1986) first suggested the idea of using expected values to predict the outcomes in a power structure. Friedkin (1992, 1993) extended the idea of expected values to analyze outcomes in an exchange network. In our analysis we use the most recent version of EVT (Friedkin, 1995). The 1995 version uses a complicated iterative algorithm in which in each step subsequently (i) exchange outcomes of each relation (v_{ij}), (ii) weight of each relation (w_{ij}), (iii) the probability that relation is used in an exchange (p_{ij}), and (iv) the probability of exclusion of both actors (d_i and d_j), are updated. In the next iteration d_i and d_j are used to compute v_{ij}. The theory assumes that both actors' claim of their share of the 24 points in their relation is increasing nonlinearly in the probability that each of them is excluded in any exchange. The algorithm stops when it converges, i.e., when all v_{ij} of two subsequent iterations are equal. See Friedkin (1995) for details of the EVT model that is used to generate the predictions.

2.3.1. Deviating predictions for EVT

The equilibrium values of v_{ij} heavily depend on i's relative probability of exclusion, d_i. That is why EVT predicts rather egalitarian payoff vectors in large seller–buyer networks. As an example, consider the S9B10 network where all sellers are connected to all buyers. EVT predicts the sellers to earn 12.2 points in their exchanges with a buyer. Again, economic theory and other exchange theories (except PDT) predict that the sellers will obtain most if not all points in these exchanges.

2.4. Network Exchange Theory (NET)

Willer and Anderson (1981) developed the basic concepts of NET. Many studies on exchange networks by more than a handful of authors were based on or motivated by NET. Willer's (1999) book '*Network Exchange Theory*' nicely summarizes this research carried out in the previous century. The book explains and relates the many different variants of NET that co-exist. Emanuelson (2005) and Emanuelson and Willer (2007) consider one of these variants "the best" (2007, p. 22). This variant is a combination of GPI-R (Lovaglia et al., 1995) and Optimal Seek Simplified (Girard and Borch, 2003). Emanuelson (2005) shows that GPI-R outperforms the other variants of NET.

The variant of NET judged to be the best can briefly be described as follows. In the first stage strong power components are identified. A *strong power component* is defined as a component containing a number of *non-excludable* (I) actors that is lower than the number of *excludable* (E) actors, where the E nodes only connect to I nodes. In the second stage network breaks are identified and deleted. After deleting network breaks the exchange network may consist of more disconnected power components. Power components can either be strong, equal, or weak. An *equal power component* is a component containing positions that are excluded at the same ratio, i.e., if all nodes are I or if all nodes are E and have the same degree (Girard and Borch, 2003, p. 232). Finally, a *weak power component* is a component that is neither strong power nor equal power (Girard and Borch, 2003, p. 228). In the final and third stage the payoff of actors in each relation is calculated after having deleted the network breaks and having identified the components of the network. The I actors in strong power components obtain 23 points in their relations with E actors who obtain 1 point. In all relations of an equal power component both actors obtain 12 points. To calculate the payoffs in weak power components the GPI-R method is employed.[3]

GPI-R integrates a position's likelihood of being included into resistance equations. Actor i's likelihood d_i of being included is calculated by assuming that actors seek exchange with all partners equally. The resistance equations are based on the principle of *equiresistance*; the agreement between two actors in a relation is assumed to occur at a point of equal resistance. The point of equal resistance occurs when

$$\frac{P_{\mathrm{A}}^{\max} - P_{\mathrm{A}}}{P_{\mathrm{A}} - P_{\mathrm{A}}^{\mathrm{con}}} = \frac{P_{\mathrm{B}}^{\max} - P_{\mathrm{B}}}{P_{\mathrm{B}} - P_{\mathrm{B}}^{\mathrm{con}}} \tag{4}$$

with P_{A} and P_{B} the payoffs of actors A and B (Emanuelson, 2005, p. 161). $P_{\max}$ and P_{con} denote the maximum and conflict payoff of both actors in this relation. Van Assen (2003) proved that (4) results in

[3] GPI-R can also be applied to the equal power components, because its application to these components yields 12 points for each actor in each relation.

the same predictions of P_A and P_B as the equidependence principle (2) of PDT, for the same values of P_{con} and P_{max}. However, PDT and GPI-R differ with respect to their calculation of P_{max} and P_{con}. In GPI-R the likelihood of being included is incorporated in P_{con} and P_{max}: $P_{i\,con} = 12(1 - d_i)$ and $P_{i\,max} = 12(2 - d_i)$. Substituting P_{max} into (4) and rearranging terms yields (1), the equation also used to derive the PDT predictions. Substituting P_{con} into (1) yields

$$P_A = 12(1 + d_B - d_A) \quad (5)$$

Of course, $P_B = 24 - P_A$.

An important step in the procedure above is the identification of possible network breaks in the second stage. A *network break* is an exchange relation in which rational actors have no interest in exchanging (Simpson and Willer, 1999, p. 273). That is, the relation is predicted not to be used at all. A network break is a *suboptimal relation*. A relation is defined to be suboptimal by Simpson and Willer (1999, p. 273) if and only if exchanging in that relation necessarily reduces the maximum number of exchanges possible for the network.[4] Not all suboptimal relations are breaks. A network break is a suboptimal relation between a position in a strong power component that is I and another position that becomes an I *after deletion of the possible break*. That is, a relation between a strong power I node and an E node can be a network break, if after deleting all network breaks this E node becomes an I node (Simpson and Willer, 1999, p. 276).

In the third stage of the procedure above a position's likelihood of being included in an exchange is calculated by assuming that 'actors seek exchange with all partners equally'. Unfortunately, this assumption is not as innocent as it seems. There are many different meaningful ways of applying this assumption resulting in different probabilities of exchange relations that are used, and consequently, probabilities of being included in an exchange. For instance, consider again the Line4. Simpson and Willer (1999) argue that the probability that BC will exchange is (1/4), and the probability that AB and CD exchange (3/4). They argue that both B and C seek each other out with probability (1/2) × (1/2) = (1/4). However, we can also convincingly argue that it is twice as likely that A and B seek each other out than B and C, because the probability that C seeks out B is only half the probability that A seeks out B (which is equal to 1). Consequently, the probability that BC exchange is only (1/4)/((1/4) + (1/2) + (1/2)) = 0.2. A third method is based upon the first iteration of Friedkin's (1995) algorithm. The probability that BC is the first exchange carried out is (1/3), which equals the total probability that BC is carried out. A fourth method to calculate the probabilities is suggested by Friedkin (1992). In this method the probabilities are calculated by summing up the number of *R*-networks in which this relation is used, divided by the number of *R*-networks. Applying this method to the Line4 results in a probability of (1/2) that the BC relation is used. Emanuelson and Willer (2007) propose to apply the Friedkin (1992) method because then probabilities can easily be calculated by hand.

In our application of NET we use the first iteration of Friedkin (1995) to calculate the probabilities of being included in an exchange. The first iteration of Friedkin considers all relations equally valuable, corresponding to the assumption 'actors seek exchange with all partners equally'. We chose the Friedkin (1995) method for several reasons. First, the method initially proposed by Simpson and Willer (1999) is not yet formalized, that is, the method was not described in sufficient detail for us to be able to implement the method. Second, we do not choose the Friedkin (1992) method because Friedkin himself suggested using another method, his 1995 method. Finally, NET researchers themselves also seem to be open to other methods to calculate the probabilities, because they have proposed to use another method than Simpson and Willer (1999).

The combination of GPI-R and Optimal Seek Simplified as explained above has not yet been implemented. Our implementation of this variant consists of the following stages, following Girard and Borch (2003) and Willer and Emanuelson (2006) as closely as possible:

(i) Indicate for each position if it is E or I.
(ii) Identify strong power components; the I nodes obtain 23 points in all their relations.
(iii) Identify network breaks and delete them.
(iv) Specify the payoffs in relations in equal and weak power components using GPI-R.

For stage (ii) a procedure of Girard and Borch (2003, p. 233) is implemented. Their procedure to identify strong power components consists of four steps following stage (i). In their first step they delete a relation between an E and an I node, if the E node also has a connection to another E node. Second, they remove relations between I nodes if one of the I nodes is connected to two or more of the E nodes. Then in step three (i) is applied again, and finally in step four it is checked if one of the resulting components satisfies the definition of strong power. The I nodes in a strong power component then obtain 23 points in all of their relations.

In stage (iii) we again use a procedure of Girard and Borch (2003, p. 237). In the first step of this procedure all suboptimal relations of an I node in a strong power position with another I node are simultaneously deleted. In step 2 of stage (iii) the nodes of the resulting network are then relabeled using (i). If there is at least one node in the previously deleted relations that has changed into an E node, then one of the relations with such a node is added back. Steps 2 and 3 are repeated until no relations are added back. Girard and Borch (2003) do not prove that the order of adding back relations does not affect the resulting network.

2.4.1. Anomalies in NET

Researchers of NET have not proven that the manipulations in the algorithm result in predictions that are in line with the theory. That is, do the algorithms of Girard and Borch (2003) in (ii) and (iii) correctly identify strong power networks and network breaks? Proving consequences of steps in an algorithm can be very difficult, but not proving the consequences can be dangerous, as we show using the Envelope network (Fig. 4). Using the Envelope we show that in some networks the algorithm advocated by researchers of NET fails and yields anomalous predictions. Applying step (i) to the Envelope reveals that only F is an I actor. Links AF, BF, DF, EF are deleted in step (ii) because therein I is linked to an E node that is also connected to another E node. After deletion, a strong power Line5 remains in which the excludable actors C and E are predicted to obtain a maximum number of points. However, C and E are excludable and will obviously earn less than strong power actors. As this example demonstrates, neither is NET "free of contradictions" (Emanuelson and Willer, 2007, p. 26) nor is it the case that

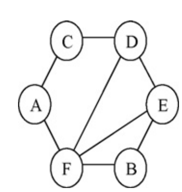

Fig. 4. The envelope network.

[4] Note that in PDT suboptimal relations are defined differently as relations in which both actors earn less than in their best alternative.

"... interactions in strong power structures are well understood ..." (Willer, 1999, p. 298).

Another problem with NET is the allocation of 1 point to an E node in a strong power network. For example, consider a full network of an even number of *N* actors all connected to each other, with one additional actor *i* linked to only one of these *N* actors. This is a weak power network. The probability that *i* is excluded will be close to 1 for large *N*. Applying (5) to such a network will yield a payoff of *i* smaller than 1, the payoff predicted for a disadvantaged actor in his relation with a strong power actor. This inconsistency is resolved if E nodes in a strong power network obtain 0, as in COT and PDT, instead of 1 point.

3. Axiomatic analysis

The limited application of exchange theories to a small set of networks has kept our theoretic understanding of behavior in exchange networks from being general. Few regularities in the predictions of network exchange theories have been established that also hold for uninvestigated networks. By investigating the satisfaction of six axioms – indicated by acronyms – by network exchange theories, we find answers to six questions:

1. ISM: Does the theory predict different relative payoffs when different amounts of profit are involved?
2. SYM: Are identically connected actors predicted to earn the same in exchange?
3. OS: Does the theory predict an outcome such that no pair of connected actors get less than the 24 points they can get by exchanging themselves?
4. LMO: Does the deletion of a link in a network always yield equal or lower payoffs to the two actors in that link?
5. EXI: Is the theory able to predict at least one payoff for every actor in every network?
6. UNI: Does the theory ever predict more than one possible payoff for any actor in any network?

The six axioms or desirable properties we consider to answer these six questions, are based on related axioms from the literature on cooperative bargaining. In this literature, scholars have identified sets of *axioms* that, when all satisfied, imply a solution to the cooperative bargaining problem originally outlined by Nash (1950). This process is called *axiomatization*. The most famous axiomatized solution is the cooperative bargaining solution of Nash (1950) and the solution concept of Kalai and Smorodinsky (1975). Nash' solution is derived from the axioms 'invariance under affine linear transformations', 'symmetry', 'Pareto optimality', and 'independence of irrelevant alternatives'. The solution of Kalai and Smorodinsky is based on the same first three axioms, but replaces the axiom of independence of irrelevant alternatives by the axiom of monotonicity.

One way of investigating the relationship between axioms and the exchange behavior they imply is to import these solution concepts. Since the sociological exchange setting considers a whole network of bilateral exchange possibilities, not just a single one, it is not straightforward to re-apply solution concepts from cooperative bargaining in this alternative context. Recently, one such attempt was made for the case of the Nash solution (Braun and Gautschi, 2006). Alternatively, we investigate what axioms the outcome predictions of already existing sociological theories of network exchange satisfy and violate. This requires redefining axioms for the sociological exchange context.

Ideally, we would proceed by formally defining "exchange outcome", defining each of the six axioms, and representing the outcome predictions of each of the four theories, and then formally demonstrating the satisfaction or dissatisfaction of each axiom by each theory. However, two of the four theories, namely EVT and NET, provide exchange outcome predictions by means of an analytically intractable computational algorithm rather than through derivation from a set of assumptions. This makes formal analysis impossible and forces us to assess the satisfaction of axioms alternatively, by inspecting the predictions for a large number of networks.

We define an *exchange outcome* as the combination of an *exchange pattern* and a *payoff vector*. The exchange pattern indicates what pairs of actors in the network exchange with one another and the payoff vector indicates what the actors obtain in their exchanges. For each exchanging pair, the payoffs add up to 24.

We modify the axiom 'invariance under affine linear transformations' from the cooperative bargaining literature to *invariance under scalar multiplication* (ISM). ISM states that predictions do not depend on the size of the profit pool.[5] If all profit pools are multiplied by a scalar, then a theory's predicted profit vector(s) should also be multiplied by that scalar.

Symmetry (SYM) means that any two actors who occupy automorphically equivalent positions (Wasserman and Faust, 1994, p. 470) in a network should be predicted to earn the same profit. A theory satisfies SYM if any predicted exchange outcome assigns identical average profit to automorphically equivalent actors. Examples of automorphically equivalent actors are all actors in the complete 4-actor network, A and C in the Line3 and the T, and A and B, and C, D, and E in the 2S3B network.[6]

Sozański (2006, pp. 422–423) considers the axiom of *outcome stability* (OS) for the network exchange setting. OS, which has also been called *dyadic rationality*, is satisfied if for any predicted outcome there exists no alternative outcome in which the sum of payoffs of two connected actors is larger than this sum in the predicted outcome. Because there is always an alternative in which two connected actors exchange and earn together 24 points, the implication is that a necessary condition for OS is that the sum of payoffs in each dyad is at least 24. OS implies the axiom of 'Pareto optimality' from the cooperative bargaining literature.

In the cooperative bargaining literature, *monotonicity* requires that adding an exchange possibility to one actor should never lead to a decrease in utility of that actor. Jackson and Wolinsky (1996) introduced the related axiom of *link monotonicity* (LMO). A theory satisfies LMO if whenever a link, and thus an exchange opportunity, is added between two actors, and the theory predicts at least one payoff vector for both the original network without the link and the new network with the added link, then the minimum payoff predicted for either actor should not decrease. In the case of EVT we checked LMO by comparing the payoffs of two connected actors in a network to these two actors' payoffs in the network without their link. If at least one of these two payoffs is increased after the deletion of their link, LMO was violated for that deleted link and

[5] In cooperative bargaining, the stronger condition of "invariance under affine linear transformations" implies that an affine linear transformation ($ax+b$, with $a>0$, where x is the actor's utility) does not affect the bargaining solution. In the context of exchange networks an actor's utility cannot be independently transformed because an exchange is defined as a constant-sum game of 24 points. Therefore, we only consider linear transformations of the common unit of profit, i.e., multiplications of all resource pools with the same constant. Note that the original property also considers the addition of a constant b to the payoff. The network exchange setting fixes the minimum earnings at zero, so that any constant b other than zero falls outside its scope. Therefore, we do not incorporate the addition of the constant in our definition of ISM.

[6] The original symmetry axiom from cooperative bargaining concerned the *utility* of the actors. Since the network exchange setting is defined in terms of *payoffs*, not utility, we instead consider identical payoff possibilities.

this network. In case of NET and PDT, for which no expected payoffs are predicted, we assessed the effect of the link deletion on their payoffs in other exchange possibilities. If the effect of the deletion of their link on at least one exchange possibility was positive for at least one actor in that deleted link, then LMO was violated for that link and this network.

The first four properties from the cooperative bargaining literature are intimately related to four principles in the sociological exchange literature. Consider the property of ISM first. The 24-point profit pool in the standard network exchange experiment was always intended to reflect an imaginary surplus. An exchange theory violating ISM predicts different exchange outcomes for a different, equally arbitrary imaginary surplus, and thereby would fail to provide an unambiguous prediction.

Now consider LMO. The idea that power derives from excludability is central to the network exchange literature. Actors who cannot be excluded have more power and obtain better exchange outcomes. When a theory violates LMO, it necessarily violates the excludability principle of exchange. Adding a link to a network gives two actors an extra relation to exchange in, making them both necessarily less excludable.

Central also is the idea that power stems from structure only. If it does, actors with structurally identical positions necessarily obtain the same expected payoff. When SYM is violated, they obtain different expected payoffs.

And fourth, OS reflects the idea that two parties should not forego an exchange opportunity if by exchanging they can both earn more than they are currently earning. This principle plays an important role in the contributions of Braun and Gautschi (2006), Sozański (2006), Bienenstock and Bonacich (1992), Bonacich and Bienenstock (1995), and Bonacich (1998) to the network exchange literature.

The two other properties of exchange theories we check are *existence* (EXI) and *uniqueness* (UNI). EXI is satisfied if a theory predicts at least one payoff vector for any network. If for some network multiple payoff vectors are predicted, UNI is not satisfied.

It is important to emphasize that, although we verify whether the four theories satisfy these properties, we do *not* expect or require them to do so. That is, we will *not* argue that a theory is bad per se if it does not satisfy one or more of these desirable properties. The purpose of the axiomatic analysis is to increase our understanding of theories of network exchange by establishing regularities in the predictions of the four theories.

The next section deals with the four theories one by one. Each theory is explained and then its satisfaction of each of the above six properties is assessed. In addition, we discuss features of the theory beyond the satisfaction of these properties.

3.1. Satisfaction of axioms by COT

The results concerning the satisfaction of the properties by COT are shown in the second column of Table 1. COT satisfies ISM. Multiplying the size of all resource pools by a scalar does not affect the categorization of the network into one of the four types. This follows from Bonacich's (1998) demonstration that the core is the solution to a linear system of rationality conditions. Multiplying the size of all resource pools by a scalar is equivalent to multiplying the vector of constants in the linear system by a scalar. The result is a multiplication of the solution vector by the same scalar.

Table 1
Four solution concepts and their satisfaction of six desirable properties

	COT	PDT	EVT	NET
ISM	Yes	Yes	No	No
SYM	No	No	Yes	?
OS	Yes	No	No	No
LMO	Yes	No	No	No
EXI	No	Yes	?	Yes
UNI	No	No	No	No

COT satisfies OS. COT requires collective rationality, namely that maximum total profit is earned. Maximum total profit can only be earned if the maximum number of exchanges is carried out, because each exchange increases total profit by 24.

COT does not satisfy SYM. For automorphically equivalent actors COT installs the same rationality restrictions on earnings. However, this does not necessarily mean that automorphically equivalent actors obtain the same payoff. They do if the network is of type 2 or type 3, but not if it is of type 4. For example, the payoff vector (6, 18, 7, 17) is one of the predicted payoff vectors by COT for A, B, C, and D in the Line4. Note that B and C are automorphically equivalent, but do not obtain the same payoff. Table 2 shows frequencies for the four types as a function of network size for all 12,112 connected networks with between 2 and 8 actors. Networks with components in category 4 have a non-unique core: 4533 + 12 + 210 + 996 = 5751 networks (47.5%) contain one or more of such components. For these networks, COT violates SYM.

COT satisfies LMO. Adding a link never decreases the minimum payoff for either of the two actors connected by that link. The link adds the coalitional rationality requirement that the actors together earn at least 24. This requirement can only raise the minimum predicted payoff of an actor.

COT does not satisfy EXI. For all networks of category 1, the core is empty. Examples are the Triangle and the Kite (Fig. 3a). Out of all investigated networks 907 + 3 + 12 = 922 (7.6%) have an empty core. For these networks, COT violates EXI.

COT does not satisfy UNI, e.g., in the Line4 and T (Fig. 1d and e). In the Line4, any allocation to A, B, C, and D satisfying $(x, 24-x, x, 24-x)$ and $0 \le x \le 24$ is in the core. If and only if a network contains a component of type 4, there are an infinite number of solutions. As Table 2 shows, there are 5751 such networks (47.5%).

3.2. Satisfaction of axioms by PDT

The results concerning the satisfaction of the six properties by PDT are shown in the third column of Table 1. PDT satisfies ISM. Multiplication of all payoffs in all relations with scalar z simply leads to a multiplication of O and C by z, and all predictions are multiplied with z.

SYM is not satisfied by PDT as Theorem 1 illustrates. The Box network (Fig. 2b), in which all actors occupy automorphically equivalent positions, has $O_A = O_C = 24 - O_B = 24 - O_D = 0$ as one of the multiple PDT solutions; note that A (C) and B (D) in general are not predicted to obtain the same payoffs.

We cannot directly verify for which networks OS is violated by PDT because PDT does not provide predictions of actor payoffs. However, indirectly we can conclude that PDT violates OS because it asserts that also suboptimal exchange relations will be used, although not frequently (Cook and Yamagishi, 1992, p. 252). Suboptimal relations occur in 11,791 networks (97.3%), hence PDT violates OS in the vast majority of networks that we investigated.

PDT violates LMO. Take for example the T network (Fig. 1e). Consider the effect of relation BD on Bs and Ds payoff. PDT predicts 24 for B in her relations with A and C, and 12 for D in her relation with C. Applying Eq. (2) to the suboptimal relation yields 18 for B and 6 for D, an equidependence of −6. Because PDT claims that this relation is used with positive probability, this relation decreases both B's and D's minimum payoff. Cook and Yamagishi's algorithm violates LMO in 4766 relations (2.8%). 1195 (9.9%) networks contain at least one such relation.

Table 2
Number of connected networks of size 2–8 containing an empty (1), strong power (2), equal power (3), non-unique (4) core component, or a combination of these components

Size	Component type(s)									
	1	2	3	4	1 and 2	1 and 4	2 and 3	2 and 4	3 and 4	All
2	0	0	0	1	0	0	0	0	0	1
3	1	1	0	0	0	0	0	0	0	2
4	0	1	1	4	0	0	0	0	0	6
5	12	7	0	0	0	0	0	2	0	21
6	3	12	25	65	0	0	0	2	5	112
7	621	142	0	0	0	6	4	80	0	853
8	270	411	4843	4463	3	6	4	126	991	11,117
All	907	574	4869	4533	3	12	8	210	996	12,112

PDT satisfies EXI. Sozański (1997) proved that a PDT solution always exists. The Cook and Yamagishi (1992) algorithm also converged always for networks up to size 8.

PDT does not satisfy UNI, as Theorems 1 and 2 in the previous paragraph demonstrate. 61 networks (0.5%) are nu-sym, which all violate UNI. UNI is also violated in the 3322 (27.4%) nu-asym networks.

3.3. Satisfaction of axioms by EVT

The results concerning the satisfaction of the six properties by EVT are shown in the fourth column of Table 1. EVT does not satisfy ISM. This can easily be demonstrated by applying EVT to the Line3. EVT predicts that each exchange occurs with a probability of 0.5. After realizing that the partners in each exchange overbid, the predicted outcome for a peripheral actor is calculated as 2.90, or 12% of the total pool. If the size of the pool is increased to 1000, then EVT expects a peripheral actor to obtain 16.30, or only 1.6% of the profit pool. It can be shown that EVT predicts that if stakes are raised (or measurement unit decreased, e.g., from English pounds to American dollars), then the share of the more powerful actor is increased.

EVT satisfies SYM because the algorithm initially allocates equal profit to every actor and subsequently applies the same operations to automorphically equivalent actors. Neither LMO nor OS is satisfied by EVT, as we can see when we compare the T network (Fig. 1e) to a disconnected 5-actor network obtained from the T by deleting relation BD. B's payoff is decreased by adding link BD, thereby violating LMO. According to EVT, relation BD is used with probability 0.07, thereby decreasing B's expected payoff from 21.1 in the Line3 to 20.7 in the T. EVT's prediction that B and D sometimes use BD violates OS, because this exchange reduces the maximum number of exchanges from 2 to 1. EVT violates OS for all but 7 networks (99.9%). That is, only in 7 networks the sum of payoffs of the two actors in each exchange relation is at least 24. LMO is violated in 25,750 relations (15.1%) for EVT, which are spread over 8458 (69.8%) networks. That is, the majority of networks contain a relation that, after removal, increases the expected payoff of at least one actor in that relation.

The low analytic tractability of EVT, in particular the three rules on how actors in a pair deal with their claims, makes formal analysis difficult. Perhaps therefore EXI has not been proven. However, a solution was obtained for all networks up to size 8, that is, the algorithm converged to one solution for each network. Of course, this is not a proof that only one solution exists or that a solution exists for all other larger networks. Note that also Friedkin (1995) does not make any statement concerning the general existence and uniqueness of his solution. In fact, Friedkin (1995) has only applied his theory to predict outcomes in one exchange network, the Kite (Fig. 3a). Consequently, the most recent version of his theory is analyzed and generalized for the first time in the present study.

EVT violates UNI. For some networks it predicts multiple exchange patterns with different associated payoff vectors. We already discussed the examples of the T and Line4 networks.

3.4. Satisfaction of axioms by NET

NET does not satisfy ISM, because it allocates 1 point to an E node in strong power networks independently of the size of the resource pool. That is, if the pool is of size 24, then the E node obtains less than 5% of the pool, but if the size is only 5, E is predicted to obtain 20% of the pool. NET thus violates ISM in all networks that contain a strong power component.

The algorithm deals with automorphically equivalent actors in the same way. However, we cannot guarantee that the algorithm satisfies SYM, because we do not know if step (iii) of the algorithm leads to a unique output. Girard and Borch (2003) do not prove that if two (or more) nodes change from I to E after deleting suboptimal relations, then the order in which these relations are added back has no effect on the final output of step (iii) of the algorithm. Perhaps a case exists where, after adding back either one of the relations, both nodes become I nodes again. Note that if a case like this exists, step (iii) has two possible outputs.

NET does not satisfy LMO, since deleting a link between two actors can increase one of these actors' payoffs. LMO is violated in 29,474 relations (17.2%), which belong to 7217 networks (59.6%). NET also violates OS in 97.3% of the networks because it predicts, just like PDT, that suboptimal links are predicted to be used sometimes. NET also violates OS in networks containing a strong power component because NET allocates only 23 points to each strong power actor, resulting in a sum less than 24 points for at least one relation in the network. Even after allocating 24 points to strong power actors and after removing breaks NET still violates OS in 11,312 networks (93.4%).

NET satisfies EXI. At least one solution always exists for the algorithm described in the previous paragraph. It is easy to verify that each of the four steps of the algorithm always generates output. NET does not satisfy UNI. For some networks, it produces multiple exchange patterns with different associated payoff vectors. For example, for the Line4, NET predicts both a 12–12 exchange in the BC relation as an exchange outcome as well as two 14.4–9.6 exchanges in the AB and CD relations.

4. Choosing laboratory networks

We argue that by strategically picking laboratory exchange networks, the statistical power needed to distinguish the predictions of theories of network exchange can be increased dramatically. We proceed by showing that for most networks of small size the predictions of the four theories hardly differ, while for a small minority differences are large. Thereafter three networks are presented that can distinguish the four theories very easily because at least two

Table 3
Average absolute differences in predicted profit out of 24 profit points

Solution concept	Solution concept		
	PDT	EVT	NET
COT	0.28	1.53	0.89
PDT		1.61 (1.79)	1.10 (1.60)
EVT			0.74 (1.23)

A number is shown for the subset of networks for which all theories have a prediction. The number between brackets is based on all networks for which the two respective theories have a prediction.

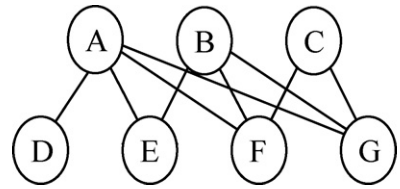

Fig. 5. A 7-actor incomplete bipartite network.

theories provide very different predictions for each of these networks.

Table 3 shows the average absolute difference in predicted profit for any pair of solution concepts. For each pair of solution concepts one can either consider all relations for which this pair has a prediction (between brackets in Table 3), or only those relations for which *all* theories have a prediction. That is, NET has no prediction for breaks and sometimes the core has no unique prediction. For 92,638 (54.4%) of all relations, all theories have predictions. The results of both types of comparisons are similar, although differences between theories are a bit smaller if only relations are considered for which all theories have predictions. Focusing on those, the relation-level profit predictions of PDT and EVT differed from one another on average by 1.61 out of 24 points. This difference was 1.10 for the pair PDT-NET and 0.74 for the pair EVT-NET, 0.28 for COT-PDT, 1.53 for COT-EVT, and 0.89 for COT-NET. Strikingly, despite the fact that COT always predicts profit 0, 12, or 24, while the other three methods can in principle predict any profit between 0 and 24, on average it does not appear to deviate from the other three more than the other three deviate from each other.

Another way to quantify differences in profit predictions is to classify exchange networks into three categories, strong-power, weak-power, and equal-power. Recall that a network is classified as equal power if all relations are equal power, as strong power if strong power but no weak-power relations exist, and as weak power if at least one relation is weak power. Because differences between payoffs 12 and 13 are small and hardly distinguishable, equal power relations were here defined as relations in which no actor earns more than 13. Relations were defined strong power if one actor earns more than 21, in weak relations one actor earns in between 13 and 21. Table 4 shows that PDT and EVT disagree on 8565 networks (70.7%), PDT and NET on 7687 networks (63.5%), and EVT and NET on 1161 networks (9.6%). The large deviations of PDT from the other two theories are largely due to the fact that it predicts perfectly equal divisions in most networks. Table 4 cross-tabulates the COT categorization with PDT, EVT, and NET's classification of networks into strong, weak, and equal power. Table 4 clearly shows how COT and PDT tend to classify networks as equal power, while EVT and NET tend to classify networks as weak power. For example, the row representing all networks categorized by the core as equal power networks (core '3') shows that all of these networks are also classified as equal by PDT, but the majority of these networks are classified as weak by EVT (98.3%) and NET (90.4%). However, the difference in the predictions for these networks is very small, yielding a low statistical power to determine the correct or best theory. Therefore, we identified three networks allowing for greater statistical power, so that less data is needed to falsify a theory.

4.1. 7-Actor incomplete bipartite network

The 7-actor incomplete bipartite network of Fig. 5 distinguishes EVT from PDT, COT, and NET. As in the S9B10 network discussed earlier, EVT predicts a much smaller payoff difference than the other theories. EVT allocates only 15.38 to strong power actor C in her relations with F and G, while all other theories allocate 23 (NET) or 24 (COT and PDT) to C.

4.2. 2S3B network

The 2S3B network distinguishes PDT, EVT, and the two other theories. The PDT algorithm predicts payoffs of 12 for all relations, while all other theories predict a clear advantage for the two sellers. NET allocates 23, COT 24, and EVT allocates on average 19.47 to each of the two sellers (and 2.76 to each buyer).

4.3. A 7-actor network

The 7-actor network of Fig. 2 distinguishes NET and EVT from PDT and COT. PDT and COT assume simultaneous exchange and therefore allocate 24 points to A, C, and E. NET and EVT, however, classify it as a weak power network with an advantage of A over C and E. EVT predicts 19.26 points for A, and 15.72 points for C and E. NET predicts that A obtains only 16.24 in an exchange with G, and 15.16 in an exchange from B and F, while C and E are predicted to obtain 14.57 from B and F. Note that EVT predicts both larger payoff differences and a larger payoff advantage of A over B and F.

Table 4
COT categories empty (1), strong power (2), equal power (3), and non-unique (4) by power type for PDT, EVT, and NET

COT	Solution concept									Total
	PDT			EVT			NET			
	Equal	Weak	Strong	Equal	Weak	Strong	Equal	Weak	Strong	
1	485	234	188	66	841	0	107	692	108	907
2	134	42	398	0	532	42	0	116	458	574
3	4,869	0	0	85	4,784	0	464	4,400	5	4,869
4	2,342	2,191	0	12	4,521	0	36	4,487	10	4,533
1 and 2	0	3	0	0	3	0	0	3	0	3
1 and 4	0	6	6	0	12	0	0	10	2	12
2 and 3	0	0	8	0	3	5	0	0	8	8
2 and 4	16	48	146	0	208	2	0	56	154	210
3 and 4	180	816	0	1	995	0	10	976	10	996
Total	8,026	3,340	746	164	11,899	49	617	10,740	755	12,112

5. Discussion

The present study evaluates four theories of exchange networks that have been applied most often, and, consequently, are well known in the field: Core Theory (COT), Power–Dependence Theory (PDT), Expected Value Theory (EVT), and Network Exchange Theory (NET). In previous research these four and also other theories were compared, but only using a small number of networks of small size and low density. Moreover, the theories were mainly compared empirically, testing the theories' predictions in the laboratory. Due to bias in the selection of test networks, it is questionable how much faith can be given to conclusions that power differences in exchange networks are well understood.

Our approach differed from the approach in previous research in that the four theories and their predictions were evaluated for all networks up to size 8 instead of the 0.2% of networks up to size 8 investigated previously. Using a formal analysis and the predictions for all these networks we (i) identified anomalies in theories of network exchange, (ii) investigated to what extent the theories satisfy some desirable theoretical properties, (iii) compared the theories' quantitative predictions and classification of networks into equal, weak, and strong, and identified three networks for which theories' predictions are very different. The results are discussed below with their implications for our understanding of exchange in networks.

COT and PDT share an assumption that leads to anomalous predictions for previously uninvestigated networks. Whereas in network exchange experiments exchanges always take place sequentially, COT and PDT assume simultaneity of exchanges. In many networks, after the first pair has exchanged, power differences shift significantly. The anomalous predictions we found could be traced back to the failure of COT and PDT to adjust for such changes in power during the network exchange process. COT and PDT predict that the 7-actor network of Fig. 2 is a strong power network because they assume simultaneity of exchange, while EVT does not.

COT and PDT can both be adjusted by replacing the simultaneity assumption by a sequentiality assumption. Here we suggest two possible adjustments of PDT. At present, Eqs. (1)–(3) of PDT are not stated in *realized* but in *potential* payoffs, i.e., payoffs given that the respective relation is realized. Working with potential payoffs allows the 24–0 solution in favor of the buyers in the complete 2S3B network. However, this is not possible if realized payoffs are taken into account because only two buyers can obtain 24 points. Consequently, the only solution is 24–0 in favor of the 2 sellers if only realized payoffs are considered. Requiring equidependence of realized payoffs makes PDT resemble the core, i.e., sometimes PDT then yields an empty solution (e.g., in the Triangle) or an indeterminate solution (e.g., in the Box). The other adjustment is a recent modification of PDT proposed by Buskens and van de Rijt (2008). In their model an outcome satisfying equidependence is calculated for a given set of probabilities that each exchange is the first to be carried out. That is, the probabilities of exchange are exogenous to the model. A byproduct of their modification is that its predictions satisfy three of the axioms we consider here, EXI, SYM, and ISM, for any given set of probabilities.

EVT predicts egalitarian outcomes in large complete bipartite networks with bipartitions that minimally differ in size. Such networks represent matching markets with slightly more buyers than suppliers. By contrast, COT, PDT, and NET, as well as Markovsky's (1987) simulation model, and standard economic theory, all predict a highly non-egalitarian outcome with most if not all of the 24 points for the seller. An example of such a network is the network of Fig. 5.

Another peculiar characteristic of the EVT solution deserves mentioning, because it is not shared by the other theories of network exchange. This characteristic is demonstrated using the 6-actor weak power network, called 'Envelope', depicted in Fig. 4. F is the only actor that is never excluded, she is predicted to obtain between 14 (in DF and EF) and 16.3 (in BF) in her exchanges. The outstanding feature of EVT's predictions is that it often predicts the presence and use of a relation that is optimal for one actor and not optimal for another. For example, EVT predicts that BF is the relation in which F can get her highest payoff (16.3 vs. 7.7 for B), but B is expected to obtain 9.8 in her relation with E. Similarly, D wants to exchange with C, but C prefers to exchange with A. Note that this state of affairs is impossible in PDT. Equidependence forces both actors in the link to have the same preference concerning exchanging in their relation.

An anomaly was detected in the algorithm of NET as well. This anomaly led to anomalous predictions in 867 (7.2%) of the networks we examined. In these networks one of the assumptions of NET that only strong power actors obtain the maximum number of points is violated. NET is thus not "free of contradictions", as Emanuelson and Willer (2007, p. 27) claim. The error in NET's algorithm can be corrected quite easily, see for example Sozański (2006, pp. 414–416).

Let us now focus on our second issue, the satisfaction of six axioms by the four theories of network exchange. Concerning UNI and EXI, EVT and NET had never been proven to satisfy them. Sozański (1997) proved EXI for PDT. Cook and Yamagishi's (1992) algorithm of PDT suggests a unique equal power solution for many networks, such as *nu-sym* and *nu-asym* networks. However, we proved that in fact an infinite number of PDT solutions exist for these networks. COT does satisfy neither EXI nor UNI. However, one could argue on both empirical and theoretical grounds that this does not represent a weakness of COT. COT suggests that if actors 'play' the 'exchange network game' repeatedly, then the exchange pattern is not stable if the core of that network is empty. Additionally, COT implies that the variance of exchange outcomes over different groups of actors playing the game is larger for networks with an indefinite core than for networks where the core is of type 2 ('equal') or type 3 ('strong'). Both these hypotheses are confirmed empirically (Bonacich and Bienenstock, 1995; Bonacich, 1998), which demonstrates that indeed *the* exchange outcome cannot be predicted for networks having an empty or indefinite solution. To conclude, EXI and UNI need not be desirable characteristics of theories.

Another desirable property we considered was SYM. The demand is merely that only structure matters. Only EVT has been shown to satisfy this property. COT and PDT both allow for allocations in which interchangeable actors earn different payoffs. We did not find such networks for NET, but we could not show NET satisfies SYM. PDT and COT both satisfy ISM, and EVT and NET do not. Although it seems reasonable to assume that the measurement unit, i.e., the size of the resource pool, should not affect the solution, there is some evidence that increasing incentives decrease the propensity of socially desirable behavior (e.g., Camerer and Hogarth, 1999, p. 33). The implication of this result for exchange networks is that if incentives or resource pool size goes up, the proportional reward for lower power actors goes down. This effect is predicted by NET (the disadvantaged actors are always predicted to obtain 1, independent of the size of the resource pool) and EVT.

OS and LMO are based on some notion of rationality. Only COT is a rational theory. All theories except COT violate OS and LMO for other networks than those with an empty core. Of course, it is an empirical issue to what extent OS and LMO are violated by human subjects exchanging in these networks. LMO has never been investigated empirically, but previous research demonstrated that OS is violated regularly. Many Pareto inefficient outcomes were observed in studies on bargaining conducted some time ago (see Roth and Malouf, 1979, p. 581). Also in exchange network experiments Pareto optimality was violated, e.g., in up to 20% of all cases B and C exchange in the Line4 network (e.g., Simpson and Willer, 1999, p. 283 report 17.5%). However, these violations were possibly the result of the design of exchange experiments. In typical experiments the exchange game was repeatedly played by the same subjects. Repeated play can affect the bargaining. For example, an exchange between B and C can be rational for B and C in an attempt to raise payoffs in future exchanges with A and D. Indeed, if the exchange game is played only once, violations of Pareto optimality and hence OS do not occur or much less frequently (Willer and Emanuelson, 2006).

Comparing the theories' predictions for all networks up to size 8 revealed some interesting general tendencies. First, COT and PDT classify many more networks as equal power than NET and EVT (respectively, 2/5 and 2/3 vs. 1/20 and 1/100), which classify most of these networks as weak. Hence, the conclusion of Willer and Emanuelson (2006, p. 6) that "Interestingly, there appear to be more weak power networks than strong or equal." is not true for PDT or COT, according to which equal power networks are predominant. The reason for this difference is that an actor's degree affects the probability of exclusion and hence the payoff in both NET and EVT. Actually, Girard and Borch (2003, p. 232) explicitly state that equal power networks are networks in which no actor is excluded or all actors have the same degree. However, the actors' degrees need not be identical for a network to be classified as equal; what counts are the restrictions on the payoffs resulting from the links.

Even though the theories' classifications were largely different, the average difference in profit split prediction over all 170,884 relations was never more than 2 points between any pair of theories. Only in a minority of cases did the predictions diverge, which demonstrated the need for careful selection of experimental exchange networks. We selected three networks out of all small networks investigated such that no pair of theories generates similar predictions for all three networks. Hence these 'critical networks' allow for 'critical tests', requiring only a relatively small number of subjects to possibly falsify a theory. Since both critical networks and anomalies in theories were found, we conclude that exchange in networks is not yet well understood. Because critical networks are networks in which different characteristics of theories result in very different predictions, investigating critical networks will also lead to an increase in the understanding of the underlying principles of how people exchange in networks.

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References

Bonacich, P., 1998. A behavioral foundation for a structural theory of power in exchange networks. Social Psychology Quarterly 61, 185–198.

Bienenstock, E.J., Bonacich, P., 1992. The core as a solution to exclusionary networks. Social Networks 14, 231–243.

Bonacich, P., Bienenstock, E.J., 1995. When rationality fails: unstable exchange networks with empty cores. Rationality & Society 7, 293–320.

Braun, N., Gautschi, T., 2006. A Nash bargaining model for simple exchange networks. Social Networks 28, 1–23.

Burke, P., 1997. An identity model for network exchange. American Sociological Review 62, 134–150.

Buskens, V., van de Rijt, A., 2008. Sequential power-dependence theory. Journal of Mathematical Sociology 32 (2), 110–128.

Camerer, C.F., Hogarth, R.M., 1999. The effects of financial incentives in experiments: a review and capital–labor–production framework. Journal of Risk and Uncertainty 19, 7–42.

Cook, K.S., Emerson, R.M., 1978. Power, equity and commitment in exchange networks. American Sociological Review 43, 721–739.

Cook, K.S., Emerson, R.M., Gillmore, M.R., Yamagishi, T., 1983. The distribution of power in exchange networks: theory and experimental results. American Journal of Sociology 89, 275–305.

Cook, K.S., Gillmore, M.R., 1984. Power, dependence, and coalitions. Advances in Group Processes 1, 27–58.

Cook, K.S., Gillmore, M.R., Yamagishi, T., 1986. Point and line vulnerability as bases for predicting the distribution of power in exchange networks: reply to Willer. American Journal of Sociology 92, 445–448.

Cook, K.S., Yamagishi, T., 1992. Power in exchange networks: a power–dependence formulation. Social Networks 14, 245–265.

Corominas-Bosch, M., 2004. Bargaining in a network of buyers and sellers. Journal of Economic Theory 115 (1), 35–77.

Edgeworth, F.Y., 1881. Mathematical Psychics. Kelley, Fairfield.

Emanuelson, P., 2005. Improving the precision and parsimony of network exchange theory: a comparison of three network exchange models. Contemporary Research in Social Psychology 10, 149–165.

Emanuelson, P., Willer, D., 2007. Testing ten theories. Unpublished manuscript.

Emerson, R.M., 1962. Power–dependence relations. American Sociological Review 27, 31–40.

Emerson, R.M., 1972a. Exchange theory, part I: a psychological basis for social exchange. In: Berger, J., Zelditch Jr., M., Anderson, B. (Eds.), Sociological Theories in Progress, vol. 2. Houghton-Mifflin, Boston, pp. 38–57.

Emerson, R.M., 1972b. Exchange theory, part II: exchange relations and networks. In: Berger, J., Zelditch Jr., M., Anderson, B. (Eds.), Sociological Theories in Progress, vol. 2. Houghton-Mifflin, Boston, pp. 58–87.

Emerson, R.M., 1976. Social exchange theory. Annual Review of Sociology 2, 335–361.

Friedkin, N.E., 1986. A formal theory of social power. Journal of Mathematical Sociology 12, 103–126.

Friedkin, N.E., 1992. An expected value model of social power: predictions for selected exchange networks. Social Networks 14, 213–229.

Friedkin, N.E., 1993. An expected value model of social exchange outcomes. Advances in Group Processes 10, 163–193.

Friedkin, N.E., 1995. The incidence of exchange networks. Social Psychology Quarterly 58, 213–221.

Girard, D., Borch, C., 2003. Optimal seek simplified. Current Research in Social Psychology 8 (16), 225–241.

Gillies, D.B., 1953. Some theorems on *N*-person games. Ph.D. Thesis. Princeton University.

Homans, G.C., 1958. Social behavior as exchange. American Journal of Sociology 62, 597–606.

Jackson, M.O., Wolinsky, A., 1996. A strategic model of social and economic networks. Journal of Economic Theory 71, 44–74.

Kalai, E., Smorodinsky, M., 1975. Other solutions to Nash's bargaining problem. Econometrica 43, 513–518.

Lovaglia, M.J., Skvoretz, J., Markovsky, B., Willer, D., 1995. Assessing fundamental power differences in exchange networks: iterative GPI. Current Research in Social Psychology 1, 8–15.

Markovsky, B., 1987. Toward multilevel sociological theories: simulations of actor and network effects. Sociological Theory 5, 100–115.

Molm, L., 1997. Risk and power use: constraints on the use of coercion in exchange. American Sociological Review 62, 113–133.

Nash, J., 1950. The bargaining problem. Econometrica 18, 155–162.

Roth, A.E., Malouf, M.W.K., 1979. Game theoretic models and the role of information in bargaining: an experimental study. Psychological Review 86, 574–594.

Shapley, L.S., 1952. Notes on the *N*-person game III: some variants of the von-Neumann–Morgenstern Definition of Solution, Rand Corporation Research Memorandum, RM-817.

Simpson, B., Willer, D., 1999. A new method for finding power structures. In: Willer, D. (Ed.), Network Exchange Theory. Greenwood, New York, pp. 270–284.

Skvoretz, J., Willer, D., 1993. Exclusion and power: a test of four theories of power in exchange networks. American Sociological Review 58, 801–818.

Sozański, T., 1997. Toward a formal theory of equilibrium in network exchange systems. In: Szmatka, J., Skvoretz, J., Berger, J. (Eds.), Status, Network, and Structure. Theory Development in Group Processes. Stanford University Press, Stanford.

Sozański, T., 2006. On the core of characteristic function games associated with exchange networks. Social Networks 28, 397–426.

Stolte, J.F., Emerson, R.M., 1977. Structural inequality: position and power in exchange structures. In: Hamblin, R.L., Kunkel, J.H. (Eds.), Behavioral Theory in Sociology. New Brunswick, Transaction Books, pp. 117–138.

Takahashi, N., Yamagishi, T., 1993. Power in social exchange networks: power–dependence theory versus elementary theory. Sociological Theory and Methods 8, 251–269 (in Japanese).

Van Assen, M.A.L.M., 2003. Exchange networks: an analysis of all networks up to size 9. Advances in Group Processes 20, 67–103.

Wasserman, S., Faust, K., 1994. Social Network Analysis: Methods and Applications. Cambridge University Press, Cambridge, UK.

Willer, D., 1999. Network Exchange Theory. Greenwood, New York.

Willer, D., Anderson, B., 1981. Networks, Exchange, and Coercion. Elsevier, New York.
Willer, D., Emanuelson, P., 2006. Looming threat or broken relation: the different functions of suboptimal relations in repeated vs. one-shot structures. Unpublished manuscript.
Willer, R., Willer, D., 2000. Exploring dynamic networks: hypotheses and conjectures. Social Networks 22, 251–272.
Yamagishi, T., Gillmore, M.R., Cook, K.S., 1988. Network connections and the distribution of power in exchange networks. American Journal of Sociology 93, 833–851.