Neighborhood Chance and Neighborhood Change: A Comment on Bruch and Mare
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Commentary and Debate

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NEIGHBORHOOD CHANCE AND NEIGHBORHOOD CHANGE: A COMMENT ON BRUCH AND MARE[1]

Elizabeth Bruch and Rob Mare in "Neighborhood Choice and Neighborhood Change" (*AJS* 112 [2006]: 667–709) are to be commended for having extended Thomas Schelling's highly stylized "thought experiment" on residential segregation to the empirical conditions observed in natural settings. Schelling demonstrated how segregation can occur even in populations that are tolerant of diversity so long as they are not outnumbered. Bruch and Mare challenge this conclusion, noting that "high levels of segregation occur only when individuals' preferences follow a threshold function" (p. 667), an assumption that they point out is not empirically plausible. On the basis of simulations with linear and empirical alternatives to Schelling's threshold function, they conclude that when "individuals are sensitive to even slight changes in neighborhood proportion own group" (p. 682) segregation largely disappears. However, we replicated their model and discovered their results were in error. Their empirical function leads to high segregation, not integration, and their linear function leads to integration only if neighborhood choice is sufficiently random that people readily move into neighborhoods where they do not

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want to live. Otherwise, linear preferences lead to more segregation than threshold preferences, the opposite of what they claim. We go further to show that sensitivity to slight changes in ethnic composition can even promote segregation in a population that not only tolerates diversity but actively seeks it.

SENSITIVITY TO CHANGE OR SENSITIVITY TO CHANCE?

Bruch and Mare made two important changes to Schelling's (1978, pp. 147–55) model:[2]

1. Schelling assumed a single tolerance threshold; Bruch and Mare increase what they refer to as "sensitivity to small changes in neighborhood composition" by using linear and empirical preference functions.
2. Schelling assumed that agents never leave a satisfactory location; their agents make chance mistakes that leave them in less-preferred neighborhoods.

Based on a comparison of segregation levels in populations with linear and threshold preference functions, Bruch and Mare conclude that "the tipping observed under the original Schelling preference function disappears when the model allows for a linear response to own-group neighborhood proportion. . . . Thus tipping only occurs under the special circumstances when individuals follow a threshold preference function" (p. 694). This is a provocative result that has quickly attracted widespread attention and recognition,[3] made all the more compelling by Schelling's receipt of the 2005 Nobel Prize. Their work demonstrates the importance of assessing the robustness of results against reasonable changes in model assumptions, especially those that increase empirical plausibility.

However, when we replicated their model, we discovered that their results were in error (a zip archive containing our "replication walkthrough" is available at the online version of *AJS*). It turns out that the low level of segregation Bruch and Mare attribute to linear preferences is possible only when neighborhood choice is sufficiently random, such that agents often move into less-preferred locations. Otherwise, segregation is even more pronounced with linear than with threshold preferences, the

[2] Bruch and Mare made several other changes as well, increasing grid size and neighborhood size and dropping Schelling's (1978) assumption that agents always move to the closest preferred location (their agents are indifferent to moving distance).

[3] The article won the 2007 Roger Gould Prize given by the *American Journal of Sociology*. It also won the Robert Park Best Article Award of the American Sociological Association Section on Community and Urban Sociology, and the Coleman Prize, awarded by the American Sociological Association Section on Rationality and Society.

opposite of what they conclude. Once the errors in their model are corrected, their model shows that Schelling's results are highly robust over alternative assumptions about ethnic preferences, including both linear and empirical functional forms.

Schelling assumed agents never act against their preferences, and Bruch and Mare are right to relax this deterministic assumption. People rarely perform a head count and therefore may have only a rough idea of a neighborhood's ethnic composition. And people may guess wrong about the direction of "changing neighborhoods." Even more important, deterministic behavior can trap a population in an unstable equilibrium that would collapse with a minor perturbation. However, the results reported by Bruch and Mare do not hold up unless we bend the stick very far the other way, as we demonstrate below.

The distinction between sensitivity to change and the effects of chance is illustrated in their five-part figure 2 (p. 676). Parts a and b depict two threshold functions that differ in the range of binary probabilities. Both threshold functions are equally insensitive to small changes in ethnic composition. However, the narrower range of probability in b indicates the weakening of ethnic preferences relative to chance, allowing a greater chance of mistakenly choosing a less-preferred neighborhood. Parts d and e depict continuous probability distributions generated by linear preference functions that are highly sensitive to small changes in ethnic composition, but random error is greater in d.

Bruch and Mare use McFadden's (1974) random utility model to implement the possibility for random mistakes. The five preference functions illustrated in their figure 2 can each be expressed using the McFadden model. In this model, k indexes all K available destinations, including the current location, j is any one of these K locations, and x_{jt} is the proportion of coethnic neighbors in location j at time t. An agent i of ethnicity l then moves to the jth vacancy at time $t+1$ with the following probability p_{ijt+1}:[4]

$$p_{ijt+1} = \frac{e^{\beta F_l(x_{jt})}}{\sum_k e^{\beta F_l(x_{kt})}}. \tag{C1}$$

The model in equation (C1) includes both a preference function and a choice function. The preference function $F_l()$ refers to the process by which the relative desirability of a location is given by the proportion x_{kt} of in-group neighbors. The choice function (eq. [C1]) refers to the process by

[4] Our figures and equations are marked *C* (for *comment*) to distinguish them from like items in Bruch and Mare's article.

which the probability p of moving to an empty location j is given by the weight β of desirability relative to random chance.

Bruch and Mare use the preference function $F_l()$ to manipulate sensitivity to change, which is either linear or threshold. The threshold preferences shown in their figure 2a and figure 2b can be implemented using equation (C1), by setting $F_l(x_{kt}) = 1$ for $x_{kt} \geq \frac{1}{2}$ and $F_l(x_{kt}) = 0$ for $x_{kt} < \frac{1}{2}$. The probability distributions in panels 2d and 2e were implemented using the linear preference function $F_l(x_{kt}) = x_{kt}$.

Through β, Bruch and Mare manipulate the weight of ethnic preference on residential choice, relative to chance. At the lower limit of $\beta = 0$, ethnic composition is irrelevant and the probability of moving into any one of K empty cells is simply $1/K$. At the other end of the spectrum ($\beta \rightarrow \infty$), the role of chance approaches zero. Now an agent will always move to and remain in its most preferred location, as assumed by Schelling.

The high level of random error in parts b and d of Bruch and Mare's figure 2 corresponds to $\beta = 1$ in (C1), which is equivalent to Bruch and Mare's equations (2) and (4).[5] According to Bruch and Mare, "A minor variation on Schelling's model is to allow for a small probability of moving into neighborhoods less than 50% own group" (p. 677). We can assess the magnitude of this probability with $\beta = 1$. Suppose half the empty cells are above and below the agent's threshold at 50% own-group. Then by their own equation (2) (or our eq. [C1] with $\beta = 1$, and a threshold functional form of $F_l()$), the "small probability of moving into neighborhoods less than 50% own group" is $e^0/(e^0 + e^1) = .27$, or about once every four moves. This probability is about as close to completely random behavior (.50 at $\beta = 0$) as to Schelling's deterministic behavior (.00 at $\beta \rightarrow \infty$).

It is important to note here that these agents are not moving into less-preferred neighborhoods because of unobserved preferences for other local attributes (such as proximity to a job or the quality of the schools). These conditions are more or less stable over time, hence agents would stay put, despite their dissatisfaction with the ethnic composition. Rather, the error term in the McFadden function allows chance deviations from ethnic preference that are unrelated to previous choices. With $\beta = 1$, it is as if people pick neighborhoods by throwing darts at a map of the city—not completely random, but with substantial room for error. Having made these errors, Bruch and Mare's agents are more likely to throw the dart again than if they had not made an error. That would not be the case if the error term corresponded to unobserved preferences that dominated a preference for coethnic neighbors.

Given the importance Bruch and Mare place on empirically calibrated preference functions, it might seem odd that they focused attention on

[5] Bruch and Mare note that the letter b in their fig. 2 represents β (p. 692).

$\beta = 1$, a value that we believe is far away from what their own empirical data would indicate. Nevertheless, while we would not characterize $\beta = 1$ as a minor variation or .27 as a small probability, we agree that it is important to know what happens as β approaches the limiting conditions. At the other extreme, their deterministic "Schelling function" in figure 2a is based on their equation (1), which is the special case of our own equation (C1) with $\beta \rightarrow \infty$ and a threshold functional form of $F_l()$. By comparing the level of segregation for threshold and linear preferences at very low and very high β, as illustrated in their figure 2, Bruch and Mare can tease out the effects of sensitivity to neighborhood change and the effects of random influences on neighborhood choice.

The results of Bruch and Mare's investigation are reported in their figure 3 (p. 681), which includes four of the five functions from figure 2. Figure 3 includes results for both the threshold functions in parts a and b of figure 2, with very low and very high β. However, they report the segregation levels for linear preferences only for $\beta = 1$. What is missing from figure 3 is the linear preference function from figure 2, part e, where $\beta = 55$. This is the crucial piece of information we need to assess the relative importance of sensitivity to change and chance mistakes.

Our figure C1 reproduces Bruch and Mare's figure 3, but with the missing results for the linear preference function from figure 2, part e. We also added one of the three empirical functions, based on the MCSUI data.[6] The results were obtained using conditions identical to those assumed by Bruch and Mare—500 by 500 grids with 15% vacancies, two ethnic groups of equal size, range 2 Moore neighborhoods, an index of dissimilarity D_t based on tracts of 100 dwellings, and running 1 million iterations from a perfectly integrated initial distribution of agents over dwellings ($D_0 = 0$).

The results in figure C1 are instructive. First, high randomness ($\beta = 1$) has a larger effect on segregation than does sensitivity to change (linear vs. threshold). After 1 million iterations, the difference in segregation between the linear and the threshold functions ranges from .11 (at $\beta = 1$) to .17 (at $\beta = 55$). In contrast, the difference in segregation between high β and the "small variation" with $\beta = 1$ is .53 for the linear function and .25 for thresholds. Second, in the near absence of random error ($\beta = 55$), the linear function in figure 2e leads to segregation, not inte-

[6] The empirical preferences are not reported in the article. We extracted MCSUI and DAS76 preferences from source code Bruch and Mare provided us on request. MCSUI agents have preferences $\beta F_l(x_{kt}) = 11.1112x_{kt} + .4341032x_{kt}^2$ for l = white and $\beta F_l(x_{kt}) = 22.3579x_{kt} - 20.26687x_{kt}^2$ for l = black. DAS76 agents have preferences $\beta F_l(x_{kt}) = 3.772429x_{kt} + 14.58365x_{kt}^2$ for l = white and $\beta F_l(x_{kt}) = 28.22248x_{kt} - 26.90168x_{kt}^2$ for l = black. DAS76 results are so similar to MCSUI results that they would overlap in fig. C1.

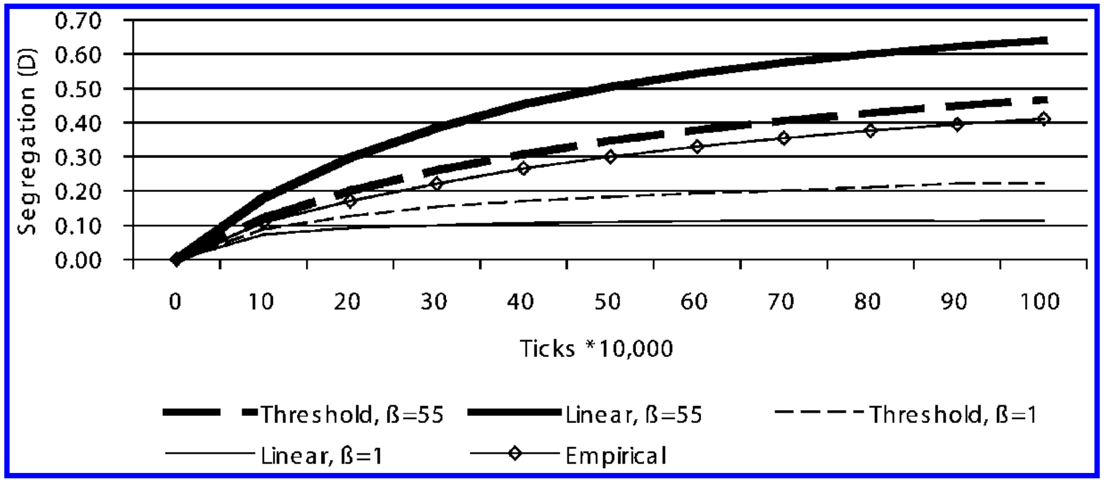

FIG. C1.—Segregation with linear, empirical, and threshold functions, under conditions identical to those in Bruch and Mare's fig. 3. The empirical function leads to segregation. Compared to thresholds, linear preferences lead to less segregation when randomness is high ($\beta = 1$) and more segregation when randomness is low ($\beta = 55$).

gration. Indeed, the level of segregation is higher for the linear function with $\beta = 55$ than for the original Schelling function. This is true as well for the empirical function. Had Bruch and Mare observed what we report in figure C1, they could not have concluded that "very high segregation occurs only when individual behavior is governed by strict thresholds" (p. 671). Once the missing condition with $\beta = 55$ is included, it is obvious that segregation occurs when individual behavior is governed by ethnic preferences and not by chance, with or without thresholds.

Although their figure 3 omits results for the linear function with $\beta = 55$, that is not because Bruch and Mare did not test this condition. In addition to $\beta = 1$, they also tested values of β "ranging from 5 to 55" (p. 692). For all these higher values of β, they found that "all of these [linear] functions produce segregation levels similar to that produced by the linear function shown in figure 2, part d," in which $\beta = 1$ (p. 692). Yet it is clearly illustrated in figure C1 that our results are not at all similar for $\beta = 55$ or for the empirical functions, for which they also report much lower levels of segregation than what we obtain.

We were also unable to find integration for the linear function with intermediate values of β, including very low values of β that Bruch and Mare did not test. Figure C2 reports results using conditions identical to those in figure C1, except for the additional values of β, which range from 0 (pure chance) to 55 (the value that approximates Schelling's determinism). For each β, we computed the level of segregation generated by threshold and linear functions. Dashes indicate the Schelling threshold function in which agents are indifferent to ethnic composition on either side of the critical value ($F_l(x_{kt}) = 1$ for $x_{kt} \geq \frac{1}{2}$ and $F_l(x_{kt}) = 0$ for $x_{kt} < \frac{1}{2}$). The solid thin line indicates a linear preference for coethnic neighbors ($F_l(x_{kt}) = x_{kt}$).

For $\beta = 0$, all moves are entirely random, leading to integration for both preference functions. For all values of β from 2 to 55 (and above), both preference functions produce values of D higher than .16, which Bruch and Mare characterize as "an intermediate level of segregation" (p. 682), and for $\beta > 3$, both functions produce segregation above the .36 that Bruch and Mare characterize as "a high level of segregation" (p. 680). It is only at $\beta = 1$ that we observe segregation with threshold preferences and integration with linear preferences. $\beta = 1$ is also unique in another way. It is the value that Bruch and Mare focus on in their article. As β approaches the lower limit, the influence of ethnic preferences becomes too weak to generate segregation for the linear function, but the threshold function still produces more than intermediate segregation, albeit below what Bruch and Mare characterize as a high level of segregation. Generalizing over the entire range of β, we find that random behavior even-

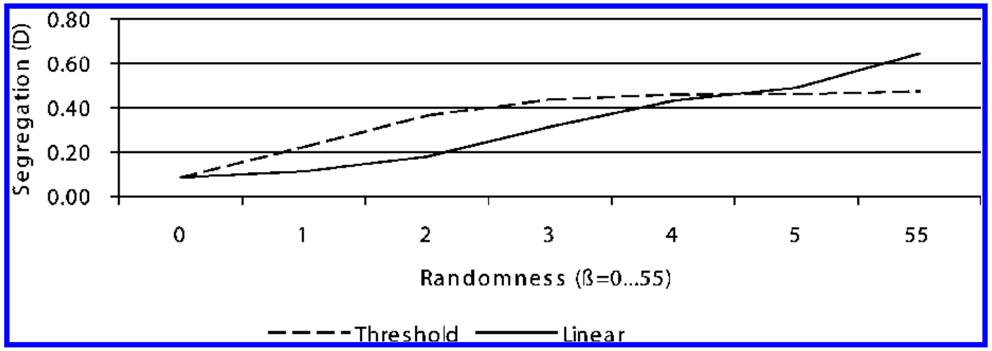

FIG. C2.—Segregation with threshold and linear preferences. All conditions are identical to those in fig. C1 except for additional values of β. As randomness decreases (β increases), segregation increases. The increase is initially steeper for threshold preferences, but eventually the linear preferences produce more segregation.

tually precludes segregation with both functional forms, but more randomness is required if preferences are threshold.

A SIMPLIFIED ANALYTICAL MODEL

There are three explanations for the discrepancy between Bruch and Mare's computational results and ours: theirs are incorrect, ours are incorrect, or both are incorrect. We began by assuming we might have implemented their model incorrectly and tried to rule out this possibility. To that end, we wrote our own code twice, using different programming languages and different programmers. Both versions gave identical results. This allowed us to minimize the chance of an unintended error of our own but does not rule out the possibility that we misunderstood their paper's verbal explanation of the model.

As an additional test, we therefore investigated the simplified model that Bruch and Mare introduce in their appendix to explain the causal mechanism that allows integration with linear but not with threshold preferences. They characterize this mechanism as a "cascade toward integration" (p. 694). They argue that with $\beta = 1$, there exists "a low probability of moving into a less desirable neighborhood (i.e., an area with few own-race neighbors)" and this in turn leads "to a net increase in own-race neighbors in that area if enough individuals are at risk of making this transition" (p. 694). Simply put, the first mistaken move makes the newly entered neighborhood slightly less undesirable to coethnics living elsewhere and slightly less desirable to the out-group neighbors already there. This in turn increases the odds that another coethnic will make the same mistake and move in. It also increases the odds that a relatively satisfied out-group member already there will make a mistake and move out. The dominoes continue to topple until all neighborhoods become integrated. Meanwhile, agents with thresholds fail to integrate. That is because the first mistaken move leaves the target neighborhood equally undesirable to coethnics and equally desirable to out-group members currently there. We agree that this is the key mechanism that yields more segregation with threshold than linear preferences at low levels of β. However, Bruch and Mare only demonstrate the cascade for $\beta = 1$ and then contend that this applies as well to the case where $\beta = 55$ (p. 694), but they offer no proof, on the grounds that "no analytic equilibrium is known for our model" (p. 699).

Nevertheless, we were able to derive β for any given equilibrium level of segregation. Their simplified model is represented by their equation (A1):

$$m[t + 1] = P(m[t])m[t],$$

where $P(m[t])$ is a matrix of transition probabilities based on McFadden's random utility model of equation (C1) and $m[t]$ is a vector with four entries. The first and second entries of $m[t]$ are the proportion black in neighborhoods 1 and 2 at time t, and the third and fourth entries are the proportion white in neighborhoods 1 and 2 at time t. Bruch and Mare consider a symmetric initial condition, $m[0] = [1\ 0\ 0\ 1]$, such that the proportion of whites in neighborhood 1 always equals the proportion of blacks in neighborhood 2. Moreover, since the proportions of blacks and whites in each neighborhood must sum to 1, any entry in $m[t]$ completely determines all three other entries. This means that the model can be reduced to a single equation in a single variable, $m_1[t]$:

$$m_1[t+1] = \frac{e^{\beta m_1[t]}}{e^{\beta m_1[t]} + e^{\beta(1-m_1[t])}}, \tag{C2}$$

which is a special case of equation (C1) with $K = 2, p_{ijt} = F_l(x_{1t}) = m_1[t]$ and $F_l(x_{2t}) = 1 - m_1[t]$. In equilibrium, the proportion black in neighborhood 1 remains constant, that is, $dm_1[t] \equiv m_1[t+1] - m_1[t] = 0$. Clearly, $m_1[t] = .5$ is an equilibrium for any value of β. We find levels of error for possible other equilibria by isolating β:

$$(1 - m_1[t])e^{\beta m_1[t]} = m_1[t]e^{\beta(1-m_1[t])} \Rightarrow \beta = \frac{ln\frac{1-m_1[t]}{m_1[t]}}{1 - 2m_1[t]}. \tag{C3}$$

The stability of all equilibria can be assessed by differentiating $dm_1[t]$ with respect to $m_1[t]$:

$$\frac{\partial dm_1[t]}{\partial m_1[t]} = \frac{2\beta e^{\beta}}{(e^{\beta m_1[t]} + e^{\beta(1-m_1[t])})^2} - 1. \tag{C4}$$

From equations (C3) and (C4) we see that $\beta = 2$ constitutes a pitchfork bifurcation point. For $0 \leq \beta \leq 2$, the only equilibrium is $m_1[t] = .5$, or perfect integration. Moreover, $\partial dm_1[t]/\partial m_1[t] \leq 0$ at these values confirms Bruch and Mare's "integration cascade" and establishes stability of the integration equilibrium. This produces the pattern for the continuous function converging toward the 50–50 distribution in Bruch and Mare's figure A1.

For $\beta > 2$, there are two additional equilibria that are stable, while $m_1[t] = .5$ is now an unstable saddle point ($\partial dm_1[t]/\partial m_1[t] > 0$).[7] From Bruch and Mare's initial condition of perfect segregation, $m[0] = [1\ 0\ 0\ 1]$, the simplified model approaches the stable equilibrium at which most blacks

[7] Note that since the only two stable equilibrium proportions in Bruch and Mare's simplified model are symmetric and yield identical levels of segregation, the ultimate level of segregation is independent of the initial level of segregation.

live in neighborhood 1, while most whites live in neighborhood 2. For example, with $\beta = 3$, the stable equilibrium proportion of blacks in neighborhood 1 is now $m_1[t] = .93$, near-perfect segregation. By setting random influence at $\beta = 1$, below the bifurcation point, Bruch and Mare found high integration in a population with linear preferences for coethnic neighbors.

THE PARADOX OF STRONG AND WEAK PREFERENCES

We have already seen how sensitivity to change facilitates integration only in populations where ethnic preferences are sufficiently weak relative to random influences on behavior. We wondered if this effect of sensitivity to change might be more robust across a wider range of β if preferences were multicultural rather than ethnocentric. In their figure 2, Bruch and Mare vary sensitivity among agents who prefer to live with coethnics. However, sensitivity to change need not be restricted to the case of ethnocentrism. Most African-Americans in the Detroit Area Studies indicate a multicultural preference for diversity (p. 683). Suppose the population has a preference for diversity instead of segregation. Is it possible that sensitivity to change can lead to integration in a multicultural population when mistakes are the exception rather than the rule?

To find out, we compared populations that differed in sensitivity to change (linear vs. threshold) in a population with a multicultural preference for diverse neighbors, as we increased β from 0 to 55. Figure C3 illustrates the multicultural functional forms.[8] The left-hand image illustrates linear and threshold preferences with minimal randomness, while the narrower range of probabilities in the right-hand image corresponds to high randomness. Notice that the functions are not perfectly symmetric. Previous research on multicultural preferences by Pancs and Vriend (2007) and by Zhang (2004) shows that this slight asymmetry (a slightly higher preference for all-in-group compared to all-out-group) is sufficient to produce the same equilibrium level of segregation that obtains with Schelling's threshold function (assuming deterministic choices). However, these studies cannot tell us if the segregation they observed was due to linear preferences (instead of threshold). The functions in figure C3 allow us to untangle the sensitivity to small changes and sensitivity to chance in a population that prefers diversity over in-group homogeneity.

[8] The multicultural threshold function is $F_l(x_{kt}) = 0$ for $x_{kt} < \frac{1}{4}$, $F_l(x_{kt}) = 1$ for $\frac{1}{4} \leq x_{kt} < \frac{3}{4}$, and $F_l(x_{kt}) = \frac{1}{4}$ for $\frac{3}{4} \leq x_{kt}$. The multicultural linear function is $F_l(x_{kt}) = 2x_{kt}$ for $x_{kt} < \frac{1}{2}$ and $F_l(x_{kt}) = 1\frac{3}{4} - 1\frac{1}{2}x_{kt}$ for $\frac{1}{2} \leq x_{kt}$. The value of $\frac{1}{4}$ is arbitrary. The results in fig. C4 do not qualitatively change for any in-group preference value above $\frac{1}{4}$, but they collapse when the value is further reduced toward zero.

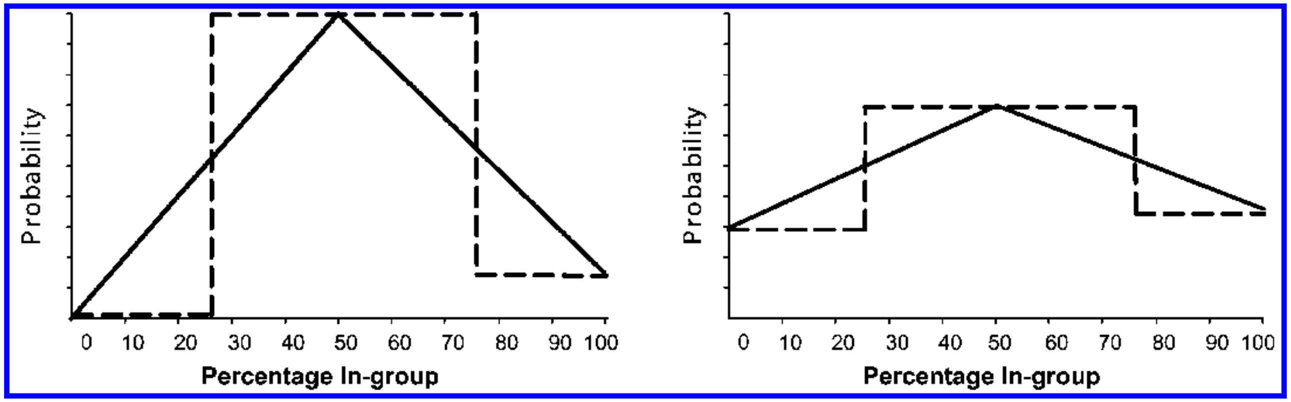

FIG. C3.—Threshold (dashed line) and linear preferences (solid line) for diversity. Randomness is higher (β is lower) in the right panel.

The results of our analysis are reported in figure C4. Inspection of the figure shows an effect of threshold preferences that is the opposite of the effect claimed by Bruch and Mare. They conclude that “high levels of segregation occur only when individuals’ preferences follow a threshold function” (p. 667). Figure C4 shows the opposite: in a multiculturalist population, tipping at the individual level can also *prevent* tipping at the population level.[9]

Comparing figures C2 and C4 reveals a fascinating paradox. On the left of figure C2 ethnocentric agents integrate, despite a universal preference for in-group neighbors. On the right of figure C4, multiculturalists segregate despite a universal preference for diversity. In both cases, individual efforts to live in a desirable neighborhood lead to an outcome that no one prefers. There is, however, a crucial difference. The ethnocentric population integrates because agents make mistakes and move to neighborhoods where they do not want to live. In contrast, the multicultural population segregates only if mistakes are sufficiently rare. And in both cases, agents must care about small changes in ethnic composition.

To sum up, Bruch and Mare conclude that segregation requires threshold preferences and that when individuals are “sensitive to small changes in race composition,” the result is “a cascade toward integration” (p. 694). “The tipping observed under the original Schelling preference function . . . only occurs under the special circumstances when individuals follow a threshold preference function” (p. 694). This is simply not true. The “special circumstance” is not a threshold function, it is that β is neither so low that both linear and threshold functions integrate (i.e., when $\beta = 0$) nor so high that both functions segregate. In between, there is a window, at the boundary of randomness, in which segregation is greater for threshold preferences. Above this window, the effect of thresholds reverses, with less segregation than observed with linear preferences. The effect can also reverse for multicultural preferences: sensitivity to small changes in ethnic composition leads to segregation, while threshold functions lead to integration.

Comparing results across both ethnocentric and multiculturalist populations, we observe the paradox of strong and weak preferences: *When ethnic preferences are sufficiently weak relative to chance, sensitivity to change can lead to greater integration in a population that prefers segregation, and when ethnic preferences are sufficiently strong, sensitivity*

[9] Fig. C4 also shows that, as β approaches determinism, segregation levels decline. However, this effect is due to nonconvergence. For comparability with Bruch and Mare’s results, we report results after 1 million iterations, but if we let simulations continue, the segregation levels continue to rise monotonically as β increases.

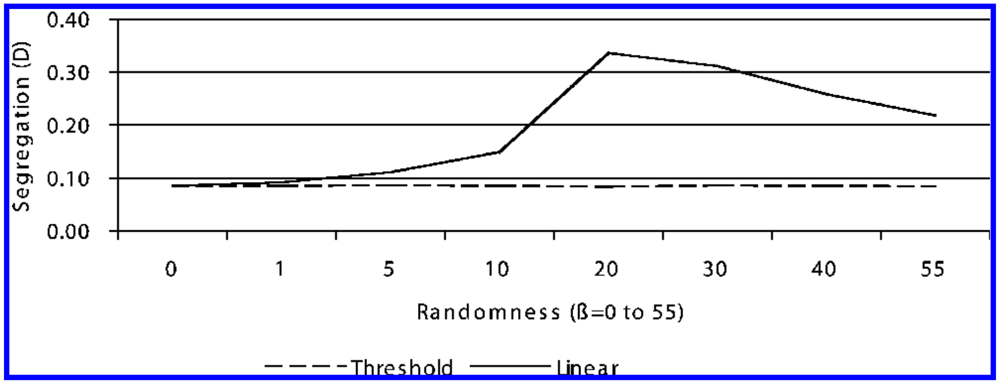

FIG. C4.—Segregation with threshold and linear preferences for diversity. All conditions are identical to those in fig. C2 except the preferences are for diversity instead of coethnic neighbors. Linear preferences lead to more segregation, compared to threshold preferences, and this difference increases as randomness decreases (β increases). The decline in segregation for $\beta > 20$ is an artifact of allowing only 1 million iterations; at convergence, segregation increases monotonically with β.

to change can lead to greater segregation in a population that prefers diversity.

In closing, we wish to underscore that the “cascade toward integration” at very high randomness is a significant discovery and an important contribution by Bruch and Mare to our understanding of the population dynamics of residential segregation. Nevertheless, the errors in their model led them to the wrong conclusion. Schelling did not overstate the tendency toward segregation. Populations with linear preferences also segregate, as do those with more empirically plausible preferences. If anything, Schelling *understated* the tendency toward segregation, which can emerge not only in a population that tolerates diversity (as Schelling demonstrated), but even among multiculturalists who actively seek diversity, so long as they are also sensitive to small changes in ethnic composition.

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